# RELATED PROBLEMS IN PARTIAL DIFFERENTIAL EQUATIONS

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Communicated by J. B. Diaz, October 18, 1967

1. **Introduction.** Let $x=(x_1, \cdots, x_n)$ and $D=(D_1, \cdots, D_n)$ where $D_i\phi(x)=\partial\phi(x)/\partial x_i$. Let $D^\alpha=D_1^{\alpha_1}D_2^{\alpha_2}\cdots D_n^{\alpha_n}$ and let $P(x, D) = \sum_{\alpha; 0\leq|\alpha|\leq m} a_\alpha(x)D^\alpha$ where $|\alpha|=\alpha_1+\cdots+\alpha_n$ and the $a_\alpha(x)$ are given functions of $x$. Finally, let $S(x)=0$ denote a cylindrical surface in $(x, t)$ space and $B(x, D)$ a nontangential boundary operator whose domain is the manifold $S(x)=0$. The smoothness required of $S(x)=0$ will depend upon the operator $B(x, D)$. We will be concerned with the following pair of initial-boundary value problems:

$$\mathrm{P}_1\begin{cases}\partial u(x, t)/\partial t = P(x, D)u(x, t), & t>0,\\ u(x, 0) = \phi(x), & \\ B(x, D)u(x, t) = f(x, t), & x\in S, t>0,\end{cases}$$

and

$$\mathrm{P}_2\begin{cases}\partial^2 v(x, t)/\partial t^2 = P(x, D)v(x, t), & t>0,\\ v(x, 0) = 0, \quad v_t(x, 0) = \phi(x), & \\ B(x, D)v(x, t) = g(x, t), & x\in S, t>0.\end{cases}$$

We assume that $B(x, D)\phi(x)$ vanishes on $S(x)=0$ and that $P(x, D)\phi(x)$ is continuous.

The interest in this paper will be in relating the solvability of $\mathrm{P}_2$ to $\mathrm{P}_1$ and conversely by means of the Laplace transform and the inverse Laplace transform. The use of the Laplace transform will necessarily impose restrictions on the choices of the functions $f(x, t)$ and $g(x, t)$, but these conditions are satisfied in a wide class of applications. By the symbolism $\mathcal{L}_s^{-1}\{\psi(x, s)\}_{s\to t^2}$ we understand the inverse Laplace transform with the variable $s$ in the transform and the variable $t^2$ in the inverted function. We then have the following results:

THEOREM 1. *If* $\mathrm{P}_1$ *is solvable with solution* $u(x, t)$ *and if*

$$g(x, t) = \Gamma(3/2)\mathcal{L}_s^{-1}\{s^{-3/2}f(x, 1/4s)\}_{s\to t^2}, \tag{1.1}$$

*then* $\mathrm{P}_2$ *is also solvable and*

$$v(x, t) = \Gamma(3/2)\mathcal{L}_s^{-1}\{s^{-3/2}u(x, 1/4s)\}_{s\to t^2} \tag{1.2}$$

*provided the inverse Laplace transform exists in* (1.1) *and* (1.2).