GROUPS OF DIMENSION 1 ARE LOCALLY FREE

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Our result is slightly more general than

THEOREM 1. *A torsion-free, finitely presented group G, with infinitely many ends, can be written as a nontrivial free product* $G_1 * G_2$.

The condition "finitely presented" can be weakened to: There is a finite complex K and a regular covering space $\tilde{K}$ with $H^1(\tilde{K}) = 0$, such that G is isomorphic to the group of covering translations of $\tilde{K}$.

From this we deduce

THEOREM 2. *If a finitely generated group G has cohomological dimension* $\leqq 1$, *then G is free* [1].

This is another way of stating the title theorem. Another consequence is

THEOREM 3. *If a finitely generated group G has a free subgroup of finite index, and if G is torsion-free, then G is free* [3].

(The references are to papers where these results have been conjectured.)

We shall indicate briefly how to prove Theorems 1 and 2. Details will appear elsewhere.

We use cohomology with coefficient group $\mathbf{Z}_2$. Ordinary cohomology is called $H^n(X)$. Cohomology with finite cochains is $H_f^n(X)$. By $\mathbf{Z}_2 G$ we denote the group ring of G with coefficient ring $\mathbf{Z}_2$; modules, projective modules, etc., are with reference to this ring; if M is a module, $M^\star$ means $\mathrm{Hom}_{\mathbf{Z}_2 G}(M, \mathbf{Z}_2 G)$.

To say that a group G has infinitely many ends, means that $H^1(G; \mathbf{Z}_2 G)$ is more than $\mathbf{Z}_2$. In terms of the regular covering space $\tilde{K}$, on which G acts freely with quotient complex K, where $H^1(\tilde{K}) = 0$, this means that $H_f^1(\tilde{K})$ contains more than two elements.

We suppose that K is a finite simplicial complex with ordered vertices; on this and on $\tilde{K}$ we have the standard cup-product of cochains defined, denoted by $\cdot$.

By a *minimal* 1-*cocycle* P we mean a finite 1-cocycle on $\tilde{K}$, which is nonzero in $H_f^1(\tilde{K})$, and which is, among all such, one involving the fewest 1-simplexes.