THE COHOMOLOGY OF PRINCIPAL BUNDLES, HOMOGENEOUS SPACES, AND TWO-STAGE POSTNIKOV SYSTEMS

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In this note, we state some results on the cohomology of topological spaces that have been obtained by a study of the Eilenberg-Moore spectral sequence. Details and proofs will appear in [**11**].

We consider either of the essentially equivalent diagrams:

Figure 1
$$\begin{tikzcd} F \arrow[r, equal] \arrow[d] & F \arrow[d] \\ E \arrow[r] \arrow[d] & Y \arrow[d] \\ X \arrow[r, "f"] & B \end{tikzcd}$$

or

Figure 2
$$\begin{tikzcd} E \arrow[d] \\ X \arrow[d, "f"] \\ B \end{tikzcd}$$

In Figure 1, $Y \rightarrow B$ is an acyclic fibration with fibre F and $E \rightarrow X$ is the fibration induced by f: $X \rightarrow B$. In Figure 2, f: $X \rightarrow B$ is a Serre fibration with fibre E. We assume (in both cases) that B is pathwise connected and simply connected. Our results concern the cohomology of E.

Let Λ be a commutative Noetherian ring. Cohomology will be taken with coefficients in Λ except where explicitly stated otherwise. We assume that $H^*(B)$ is Λ-flat and that $H^*(X)$ and $H^*(B)$ are of finite type as Λ-modules. Then there is a spectral sequence of differential Λ-algebras $\{E_r\}$, defined by Eilenberg and Moore [**6**], which satisfies the conditions:

(i) $E_2 = \mathrm{Tor}_{H^*(B)}(\Lambda, H^*(X))$, where $H^*(X)$ has the structure of left $H^*(B)$-module determined by the map f^*: $H^*(B) \rightarrow H^*(X)$, and

(ii) $\{E_r\}$ converges to $H^*(E)$, in the sense that E_∞ is isomorphic to the associated graded algebra $E^0H^*(E)$ of $H^*(E)$ with respect to a suitable filtration.

With these hypotheses and notations, we have the following result.

THEOREM. *Let $H^*(B)$ be a polynomial algebra, and let X be one of the following:*

(a) *$X = BG$, where G is a compact connected Lie group such that $H^*(BG)$ is a polynomial algebra on even degree generators.*

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