# THE SOLUTION OF BOEN'S PROBLEM

BY ERNEST SHULT

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A finite $p$-group $P$ is said to be $p$-automorphic if and only if it admits a group of automorphisms $G$ which transitively permutes its elements of order $p$. A standing problem has been the proof of

$C_1$. *$p$-automorphic $p$-groups of odd order are abelian.*

A number of authors have proved special cases of $C_1$ as well as special cases of more general propositions [1, 2, 3, 5, 6, 7, 8]. Both $C_1$ and all of the generalizations of it which have been considered in the literature follow from Theorem 1 which appears below.

In [2] it is observed that if $P$ is a smallest counterexample to $C_1$, then there is associated with $P$, an anticommutative (not necessarily associative) algebra $A$ over $GF(p)$, whose dimension coincides with the number of elements in a minimal generating set of the $p$-automorphic group $P$. Further, if $G$ is the hypothesized group of automorphisms of $P$, then $G$ also acts as a group of automorphisms of $A$ in such manner that both $A$ and the Frattini-factor group of $P$ are isomorphic as $GF(p)G$-modules. Accordingly, Kostrikin [6] has introduced the notion of *homogeneous algebra*, i.e. a finite dimensional algebra $A$ over a finite field $GF(q)$, which admits a group of automorphisms $G$, transitively permuting its nonzero elements. Such algebras enjoy two basic properties: ($P_1$) if $q$ is odd, they are anticommutative [6], and ($P_2$) left multiplication by an element induces a nilpotent transformation of $A$ [2]. Then $C_1$ is a consequence of the proposition:

$C_2$. *If $A$ is an homogeneous algebra of odd characteristic then $A^2=0$.*

One may also define semi-$p$-automorphic $p$-groups (spa-groups) as finite $p$-groups admitting a group of automorphisms $G$ which is transitive on the cyclic subgroups of order $p$. This carries with it the corresponding notion of *spa-algebra*, i.e. an anticommutative finite dimensional algebra $A$ over $GF(q)$, admitting a group of automorphisms $G$ transitive on the 1-dimensional subspaces of $A$. (Property $P_2$ holds for such an algebra, but $P_1$ must be hypothesized if $q$ is exceeded by the dimension of $A$.) The following two conjectures have been considered in [3, 7, 8]: