EXAMPLES OF NONTRIVIAL H-COBORDISMS

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Farrell and Hsiang have shown that ∃ an h-cobordism W with ends M and M_1 which are not homeomorphic [11]. Thus W is not topologically trivial, i.e., is not homeomorphic to $M \times I$. The purpose of this note is to show that if $n \geq 6$ and M^{n-1} is the boundary of a regular neighborhood X^n of a 2-complex, and M^n is any p.l. h-cobordism which has M as one end, then W is homeomorphic to $M \times I$ iff the Whitehead torsion $\tau(W, M) = 0$. Thus W is topologically trivial iff it is p.l. trivial. A corollary is that for any finitely generated group π with $\mathrm{Wh}(\pi) \neq 0$, ∃ an h-cobordism W with $\pi = \pi_1(W)$ which is not topologically trivial.

Thus these results do not contain, nor are they contained in, the results of Farrell and Hsiang. Their approach is partly algebraic in that certain special facts about Whitehead groups are used. The approach here is geometric, and is based upon the study of "geometric groups" [7].

NOTATION. If K is a finite connected complex, an abstract regular neighborhood X^n of K is a compact p.l. n-manifold containing K as a subcomplex such that X collapses to K. If B is a metric space and $A \subset B$, the statement that B δ-deforms to A means ∃ $h: B \times I \to B$ with

$$h(x, 0) = x, \qquad x \in B,$$
$$h(x, t) = x, \qquad x \in A, t \in I,$$
$$h(x, 1) \in A, \qquad x \in B,$$

and the diameter of $h(x, I)$ is less than δ for all $x \in B$. In other words, A is a δ-deformation retract of B.

The following theorem is the basis for this note, and is Corollary 3 of Theorem 2 of [7].

THEOREM 1. HYP. Y is a p.l. n-manifold and $L \subset Y$ is a connected subcomplex with dim $L \leq 2$.

CONCLUSION: ∃$\delta > 0$ such that if V is any finite complex and $h: V \to Y$ is a topological embedding such that $h \mid h^{-1}(L): h^{-1}(L) \to L$ is p.l. onto and such that $h(V)$ δ-deforms to L, then $h^{-1} \mid L: L \to V$ is a simply homotopy equivalence.