DISCONJUGATE nth ORDER DIFFERENTIAL EQUATIONS AND PRINCIPAL SOLUTIONS[1]

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Consider the nth order linear differential equation

$$(0.1) \qquad P_n(D)[x] = 0, \qquad D[x] = dx/dt = x',$$

where $P_n(D) = P_n(t, D)$ and

$$(0.2) \qquad P_n(t, \lambda) = a_n(t)\lambda^n + \cdots + a_0(t), \quad a_n(t) > 0,$$

is a polynomial with real-valued, continuous coefficients on a t-interval I. §1 deals with disconjugacy criteria for (0.1). §2 deals with the existence of "principal" solutions for a disconjugate equation (0.1), as well as with the existence of solutions having specified estimates for their logarithmic derivatives. Proofs and related results will appear elsewhere.

1. Disconjugacy criteria. The differential equation (0.1) is said to be disconjugate (cf. [**9**], $n=2$) on I if no solution ($\not\equiv 0$) has n zeros, counting multiplicities, on I. If $u_1, \cdots, u_k$ are of class $C^{k-1}(I)$, we shall denote their Wronskian by $W(u_1, \cdots, u_k) = \det(D^{i-1}[u_j])$, for $i, j = 1, \cdots, k$. In particular, $W(u_1) \equiv u_1$.

DEFINITION. A set of functions $u_1, \cdots, u_{n-1}$ of class $C^n(I)$ is said to have property W (Pólya [**7**]) or to be a $w_n(I)$-system if

$$(1.1) \qquad W(u_1, \cdots, u_k) > 0 \quad \text{for } k = 1, \cdots, n-1.$$

DEFINITION. A set of functions $u_1, \cdots, u_{n-1}$ of class $C^n(I)$ is said to be a $W_n(I)$-system if, for $k=1, \cdots, n-1$ and all sets of indices $(1 \leqq) i(1) < \cdots < i(k) (\leqq n-1)$,

$$(1.2) \qquad W(u_{i(1)}, \cdots, u_{i(k)}) > 0 \text{ on } I,$$

or, equivalently,

$$W(u_j, u_{j+1}, \cdots, u_k) > 0 \quad \text{for } 1 \leqq j \leqq k \leqq n-1.$$

In particular, (1.2) implies that $u_k > 0$ and that

$$(1.3) \qquad u_1'/u_1 < \cdots < u_{n-1}'/u_{n-1}.$$

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