GLOBAL RATIO LIMIT THEOREMS FOR SOME NONLINEAR FUNCTIONAL-DIFFERENTIAL EQUATIONS. II

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1. Introduction. A previous note [1] introduced some systems of nonlinear functional-differential equations of the form

$$\dot{X}(t) = AX(t) + B(X_t)X(t-\tau) + C(t) \quad t \geqq 0, \tag{1}$$

where $X=(x_1, \cdots, x_n)$ is nonnegative, $B(X_t)$ is a matrix of nonlinear functionals of $X(w)$ evaluated at all past times $w \in [-\tau, t]$, and $C=(C_1, \cdots, C_n)$ is a nonnegative and continuous input function. Some global ratio limit theorems were then stated for one of these systems. Here two other cases are considered. In particular, we study the dependence of the stability properties of (1) on the time lag τ.

Our systems are defined as follows. Given any positive integer n; any real numbers α, u, $\beta>0$, and $\tau \geqq 0$; and any $n \times n$ semistochastic matrix $P=\|p_{ij}\|$ (i.e., $p_{ij} \geqq 0$ and $\sum_{k=1}^{n} p_{ik}=0$ or 1), let

$$\dot{x}_i(t) = -\alpha x_i(t) + \beta \sum_{k=1}^{n} x_k(t-\tau) y_{ki}(t) + C_i(t), \tag{2}$$

$$y_{jk}(t) = p_{jk} z_{jk}(t) \left[\sum_{m=1}^{n} p_{jm} z_{jm}(t) \right]^{-1}, \tag{3}$$

and

$$\dot{z}_{jk}(t) = [-u z_{jk}(t) + \beta x_j(t-\tau) x_k(t)]\theta(p_{jk}), \tag{4}$$

for all $i, j, k = 1, 2, \cdots, n$, where

$$\begin{aligned} \theta(p) &= 1 \quad \text{if } p > 0, \\ &= 0 \quad \text{if } p \leqq 0. \end{aligned}$$

The initial data in $[-\tau, 0]$ is always chosen continuous, nonnegative, and with $z_{jk}(0)>0$ iff $p_{jk}>0$.

In Grossberg [1], we announced some results for the case

$$P = \begin{pmatrix} 0 & \frac{1}{n-1} & \frac{1}{n-1} & \cdots & \frac{1}{n-1} \\ & & 0 & & \end{pmatrix}.$$