MULTIPARAMETER SPECTRAL THEORY

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1. Introduction. On an occasion calling for a topic of general mathematical interest, I owe my audience some explanation for having chosen one of the less known areas of analysis as my subject today. The idea of considering eigenvalue problems with several parameters occurs quite naturally in certain boundary-value problems, in particular those which lead to the Mathieu functions; the area in question is thus not unknown to the applied mathematician or the specialist in the "special functions of analysis." However, the exploitation of this area in the spirit of classical analysis, in particular for ordinary differential equations, has reached no more than a preliminary stage. The same is true when we examine this area in the spirit of linear algebra or again of functional analysis. It is when we look at this area in a more abstract way that another justification appears for recommending it as worthy of general interest. The formal theory of eigenvalue problems with several parameters makes extensive use of algebraic ideas which, though not new, have gained great vogue in recent years.

The spectral theory of linear operators has tended to be concentrated to an overwhelming degree on the study of endomorphisms, vis-à-vis the identity. In what I shall refer to as the standard case, one has a linear space G, usually over the complex field, and a linear operator A on G into itself. In the spectrum, we study the nature of the linear combination $A-\lambda I$, where I is the identity, and λ a complex scalar. This has, of course, been the subject of an enormous literature and has applications too numerous to mention.

Here I am concerned with two generalisations, neither particularly new, but only now beginning to claim their due share of attention. In one of these we allow nonlinear dependence on the scalar parameter, in particular, polynomial or rational dependence. In the matrix context, this is the topic of λ-matrices (see e.g. [67]). For the differential equations context, there is early work of R. E. Langer [64], [65] and a good deal of more recent work [1], [35], [58], [60], [63], [73]. In the operator context, work of a more spectral character includes [60], [63], [68], [73].

The second generalisation is that in which we introduce several

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