PROOF. Let $G=\sum\{G_n \mid n\in J\}$ where G_n is solvable of radical class n. Then $G\in\mathfrak{G}$ and has radical class ω. Let $H=\prod\{H_k \mid k\in J, H_k\simeq G\}$. H has a subgroup satisfying the hypothesis of Theorem 3. Hence $H\notin\mathfrak{L}$. Consequently, $H\notin\mathfrak{G}$.

Classes of groups satisfying the conditions of Theorems 4 and 5 include the classes SN^*, SI^*, subsolvable and polycyclic.

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ALGEBRAIZATION OF ITERATED INTEGRATION ALONG PATHS[1]

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If Ω is the vector space of C^∞ 1-forms on a C^∞ manifold M, then iterated integrals along a piecewise smooth path α: $[0, l]\to M$ can be inductively defined as below:

For $r\geqq 2$ and $w_1, w_2, \cdots, \in\Omega$,

$$\int_\alpha w_1\cdots w_r=\int_0^l\left(\int_{\alpha^t}w_1\cdots w_{r-1}\right)w_r(\alpha(t),\dot\alpha(t))dt$$

where $\alpha^t=\alpha\mid[0,t]$. (See [3].)

This note is based on the following algebraic properties of the iterated integration:

(a) $(\int_\alpha w_1\cdots w_r)(\int_\alpha w_{r+1}\cdots w_{r+s})=\sum\int_\alpha w_{\sigma(1)}\cdots w_{\sigma(r+s)}$ summing over all (r,s)-shuffles, i.e. those permutations σ of $\{1,\cdots,r+s\}$ with $\sigma^{-1}(1)<\cdots<\sigma^{-1}(r)$, $\sigma^{-1}(r+1)<\cdots<\sigma^{-1}(r+s)$.

(b) If $p=\alpha(0)$ and if f is any C^∞ function on M, then

$$\int_\alpha fw=\int_\alpha(df)w+f(p)\int_\alpha w.$$

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