## A LIE PRODUCT FOR THE COHOMOLOGY OF SUBALGEBRAS WITH COEFFICIENTS IN THE QUOTIENT

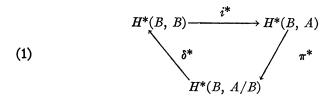
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1. Outline. We consider an algebra (i.e. an associative algebra or a Lie algebra) A and a subalgebra B. Then B, A and also A/B are (two-sided) B-modules in the obvious fashion. The exact sequence of coefficient modules

$$0 \longrightarrow B \xrightarrow{i} A \xrightarrow{\pi} A/B \longrightarrow 0$$

induces on the (graded) Hochschild [resp. Eilenberg-Mac Lane] cohomology modules the exact triangle of homomorphisms



The product operation in B, and similarly in A, induces a graded Lie algebra (GLA) structure (here called the *cup structure*) on  $H^*(B, B)$  and  $H^*(B, A)$  (cf., e.g., Gerstenhaber [2], Nijenhuis and Richardson [6]), and  $i^*$  is known to be a homomorphism of these structures. The cup structure on  $H^*(B, B)$  is abelian; cf. [2]. It is also known that  $H^*(B, B)$  has another GLA structure (here called the *comp structure*) with respect to the reduced grading (elements of  $H^n(B, B)$  have reduced degree n-1; cf. [2], [7]). The following theorem supplements this information.

THEOREM. Let A be an algebra, B a subalgebra and let A/B have its natural structure as a B-module. Then  $H^*(B, A/B)$  has a GLA structure (cup structure). The maps  $i^*$  and  $\pi^*$  in the exact triangle (1) are homomorphisms of cup structures. The image of  $i^*$  belongs to the center of  $H^*(B, A)$ . The map  $\delta^*$  is a homomorphism between the cup structure of  $H^*(B, A/B)$  and the comp structure of  $H^*(B, B)$ .

The theorem has immediate relevance for the theory of deformations.  $H^1(B, A)$  is the set of infinitesimal nontrivial deformations of

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