A LIE PRODUCT FOR THE COHOMOLOGY OF SUBALGEBRAS WITH COEFFICIENTS IN THE QUOTIENT

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1. Outline. We consider an *algebra* (i.e. an associative algebra or a Lie algebra) A and a subalgebra B. Then B, A and also A/B are (two-sided) B-modules in the obvious fashion. The exact sequence of coefficient modules

$$0 \rightarrow B \xrightarrow{i} A \xrightarrow{\pi} A/B \rightarrow 0$$

induces on the (graded) Hochschild [resp. Eilenberg-Mac Lane] cohomology modules the exact triangle of homomorphisms

$$\text{(1)} \qquad \begin{tikzcd} H^*(B, B) \arrow[rr, "i^*"] & & H^*(B, A) \arrow[dl, "\pi^*"] \\ & H^*(B, A/B) \arrow[ul, "\delta^*"] & \end{tikzcd}$$

The product operation in B, and similarly in A, induces a graded Lie algebra (GLA) structure (here called the *cup structure*) on $H^*(B, B)$ and $H^*(B, A)$ (cf., e.g., Gerstenhaber [2], Nijenhuis and Richardson [6]), and i^* is known to be a homomorphism of these structures. The cup structure on $H^*(B, B)$ is abelian; cf. [2]. It is also known that $H^*(B, B)$ has another GLA structure (here called the *comp structure*) with respect to the reduced grading (elements of $H^n(B, B)$ have reduced degree $n-1$; cf. [2], [7]). The following theorem supplements this information.

THEOREM. *Let A be an algebra, B a subalgebra and let A/B have its natural structure as a B-module. Then $H^*(B, A/B)$ has a GLA structure (cup structure). The maps i^* and π^* in the exact triangle* (1) *are homomorphisms of cup structures. The image of i^* belongs to the center of $H^*(B, A)$. The map δ^* is a homomorphism between the cup structure of $H^*(B, A/B)$ and the comp structure of $H^*(B, B)$.*

The theorem has immediate relevance for the theory of deformations. $H^1(B, A)$ is the set of infinitesimal nontrivial deformations of

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