COMPACTIFICATION OF STRONGLY COUNTABLE DIMENSIONAL SPACES[1]

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In this paper all spaces, including compactifications, are separable metrizable. Recall the following definitions. A space X is strongly countable dimensional if X is a countable union of closed finite-dimensional subsets. X is a G_δ space if X is a G_δ-set in each space in which it is topologically embedded. A space Y is a pseudo-polytope if $Y=\Sigma_1\cup\Sigma_2\cup\cdots$, where each Σ_i is a simplex, $\Sigma_i\cap\Sigma_j$ is either empty or a face of both Σ_i and Σ_j, and diam $\Sigma_i\to 0$ as $i\to\infty$. The term map always denotes a continuous function. Other notation is as in [3] and [8].

In [5] Lelek proved that every G_δ-space X has a compactification dX such that $dX\backslash X$ is a pseudo-polytope. He then raised the question of whether every strongly countable dimensional G_δ space X has a strongly countable dimensional compactification. This paper answers that question in the affirmative. We first state some preliminary propositions.

PROPOSITION 1. *Let $M\subset X$ with* $\dim M\leq n$, *and let* $\{U_i \mid i=1,2,\cdots\}$ *be a sequence of sets open in X and covering M. Then there is a sequence* $\{V_i\mid i=1,2,\cdots\}$ *of sets open in X and covering M such that* $\operatorname{ord}\{V_i\mid i=1,2,\cdots\}\leq n+1$ *and such that* $V_{k(n+1)+j}\subset U_{k+1}$ *for* $k=0,1,2,\cdots$ *and* $j=1,2,\cdots,n+1$.

PROOF. The proof involves only a slight extension of the argument on page 54 of [2].

PROPOSITION 2. *Let G be an open subset of a totally bounded space Y, and let $M_1, M_2, \cdots, M_r$ be relatively closed subsets of G with* $\dim M_i=m_i<\infty$ *for $i=1,2,\cdots,r$. Let $\epsilon>0$. Then there is a collection* $\{G_i\mid i=1,2,\cdots\}$ *such that* $G=\bigcup_{i=1}^{\infty} G_i$ *and*

(i) *Each G_i is open in Y.*

(ii) $\{G_i\mid i=1,2,\cdots\}$ *is star-finite.*

(iii) $\overline{G}_i\subset G$ *for* $i=1,2,\cdots$.

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