LOCALLY COMPACT SEMILOCAL RINGS

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We seek to determine when a locally compact ring is the topological direct product of finitely many algebras over locally compact fields; here we shall consider only commutative rings with identity.

A *semilocal* ring is a commutative ring with identity that possesses only finitely many maximal ideals. A *local* ring is a commutative ring with identity that possesses only one maximal ideal (in particular, we do not require that a local ring be noetherian nor that the intersection of the powers of its maximal ideal be the zero ideal). An *equicharacteristic* ring is a commutative ring with identity A such that for every maximal ideal $\mathfrak{m}$ of A, $A/\mathfrak{m}$ has the same characteristic as A. An algebra over a field is a *Cohen* algebra if it is a local algebra whose maximal ideal has codimension one.

THEOREM 1. *If A is a commutative Hausdorff topological ring with identity, then A is a locally compact equicharacteristic semilocal ring none of whose maximal ideals is open if and only if A is the topological direct product of finitely many finite-dimensional Cohen algebras over indiscrete locally compact fields that have the same characteristic.*

OUTLINE OF PROOF. The condition is clearly sufficient. In proving that the condition is necessary, we establish first that the invertible elements of A form an open set, that inversion is continuous where defined, and consequently that every maximal ideal of A is closed. For this, theorems of Kaplansky [**3**, Theorem 6, Lemma 3] enable us to assume that A is totally disconnected; Kaplansky's characterization of compact semisimple rings [**2**, Theorem 16] yields the result in this case.

Next, we consider the case where A is totally disconnected (if A has prime characteristic, the theory of characters implies that A is necessarily totally disconnected). If A has characteristic zero, then A contains a subfield Q algebraically isomorphic to the field of rationals; the open additive subgroups of Q form a fundamental system of neighborhoods of zero for the induced topology on Q, which is indiscrete. Similarly, if A has prime characteristic, then A contains a field $P(a)$ algebraically isomorphic to the field of rational functions over a finite field P; the open $P[a]$-submodules of $P(a)$ form a fundamental system of neighborhoods of zero for the induced topology on $P(a)$, which is indiscrete. Consequently in both cases we may apply

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