THE OBSTRUCTION TO FIBERING A MANIFOLD OVER A CIRCLE

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1. **Introduction.** In [1], Stallings considers the following question. When does a 3-manifold fiber a circle? Browder and Levine generalized Stallings' result to differentiable and piecewise linear manifolds M of dimension greater than five under the restriction that $\pi_1(M) \cong Z$. Their theorem is purely *homotopic* in nature. That is if $h: M' \to M$ is a homotopy equivalence and $f: M \to S^1$ is a smooth fiber map then there always exists a smooth fiber map $f': M' \to S^1$ such that f' is homotopic to foh.

This result is *false* if we drop their restriction on the fundamental group. In particular let N be the cartesian product of a 3-dimensional lens space L with fundamental group Z_{p^2} and the torus T^{n-3} where $n \geqq 5$. Let $M = N \times S^1$ and $f: M \to S^1$ denote projection onto the second factor. Then there exists a manifold M' and a homotopy equivalence $h: M' \to M$ (in fact we may take M' to be h-cobordant to M) such that a smooth fiber map $f': M' \to S^1$ homotopic to $f \circ h$ cannot exist. This example is based on recent deep results of Bass and Murthy [3] concerning the structure of the projective class group. In a joint paper with W. C. Hsiang [4] we use this example to construct an *h-cobordism* (W, M, M') *which is not homeomorphic to* $M \times [0, 1]$.

In this paper we will state necessary and sufficient conditions, in terms of a new obstruction theory, for a manifold M^n $(n \geqq 6)$ to fiber a circle. No restrictions will be placed on the fundamental group of M. We will always work in the differential category, but the corresponding theorem is also true in the piecewise-linear category.

2. **Description of obstructions.** Let M^n be a closed connected smooth manifold with $n \geqq 6$. Let $f: M \to S^1$ be a continuous map. (Recall that the homotopy class of f is an element of $H^1(M, Z)$.) We will state three properties about f which are necessary and sufficient to guarantee the existence of a smooth fiber map $\bar{f}: M \to S^1$ homotopic to f. For convenience we restrict our attention to maps f such that $f_\#: \pi_1(M) \to \pi_1(S^1)$ is onto. (This is equivalent to considering only indi-

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