THE ADAMS SPECTRAL SEQUENCE FOR $U^*(X, Z_p)$ AND APPLICATIONS TO LIE GROUPS, ETC.

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1. **Preliminaries.** In [1] the structure of the weakly complex *bordism* of 1 connected semisimple Lie groups was studied via the Milnor, Eilenberg-Moore, Rothenberg-Steenrod sequence. See [1] for notation. In this paper we amplify the Adams spectral sequence [2], [3], [4] and relate this tool to the weakly complex cobordism theory. The techniques apply to any finite CW complex. In particular we apply them to real projective spaces and to 1 connected compact semisimple Lie groups.

As in the bordism theory [1], it is useful to introduce coefficients into the cobordism theory. Z_p coefficients arise via [5]. Let

$$\Lambda_p = U^*(pt, Z_p) = Z_p[Y_1, Y_2, \cdots] \dim Y_i = -2i, \qquad i \geqq 1$$

and define $\Lambda_p[1/Y_{p-1}] = \text{direct lim } 1/Y_{p-1}^n \Lambda_p$. $\Lambda_p[1/Y_{p-1}]$ is the ring obtained from Λ_p by making Y_{p-1} a unit. $\Lambda_p[1/Y_{p-1}]$ coefficients can be introduced. $U^*(X, \Lambda_p[1/Y_{p-1}])$ denotes the resulting theory.

The techniques of this paper allow us to extend the theorems in [1]. For example:

THEOREM 1. *Let K be a* 1 *connected compact semisimple Lie group and p a prime. Then $U^*(K, \Lambda_p[1/Y_{p-1}])$ is an exterior algebra over the coefficient ring $\Lambda_p[1/Y_{p-1}]$ generated by rank K elements (except possibly for $U^*(K, \Lambda_2[1/Y_1])$ where K contains E_7 or E_8 as a factor). See* [1, *Theorem* 2].

We intend to make further applications in the detailed version of this paper and remove the "except possibly" statement in the above theorem.

2. **The setting.** Let $\mathfrak{J}$ denote the category of CW complexes having only finitely many cells in each dimension and maps between such spaces. A spectrum X consists of an integer N and spaces $X_i \in \mathfrak{J}$, $i \geqq N$, together with an explicit imbedding $SX_i \to X_{i+1}$. Given two spectra X and Y, a map $f: X \to Y$ is an integer $M \geqq 0$ and maps $f_i: X_i \to Y_i$, $i \geqq M$, commuting with suspensions in the obvious way. A homotopy h between f and g is an integer M' and homotopies h_i