

IMBEDDING THE STABLE HOMOTOPY CATEGORY

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The stable homotopy categories have attracted the attention of topologists for more than a decade. The most basic such category, $\mathcal{C}\mathcal{W}sh$, is the category of finite CW-complexes and stable (under suspension) homotopy classes of maps. As a category, it has many unusual properties: it is non-Abelian, although sums agree with products; every object is projective, etc. There is a sizeable literature (see [1], [2], [3], [5], and [6]).

However, a general structure theorem for this category is lacking. A full algebraic description of the category seems out of reach, because it would describe the stable homotopy of spheres, for example. But we may ask for a description modulo some basic geometry. It appears to the author that, for the present, an ideal description would consist of some algebraic structure defined in terms of the stable homotopy ring of spheres, G_* (see [7]).

We announce here an imbedding of the category $\mathcal{C}\mathcal{W}sh$ in a quotient of the category of exact couples of G_* -modules. Details of these results, and the generalizations of §3, will appear in future publications.

1. Let $\mathcal{E}\mathcal{C}_R$ be the category of bigraded exact couples (4) of modules, over the graded ring R ($1 \in R^0$). R acts on the second summand of such a module via maps

$$R^s \times A^{p,q} \rightarrow A^{p,q+s}.$$

We assume $A^{p,q} = 0$, if $p < 0$; $B^{p,q} = 0$, if $q < 0$. A morphism in this category is a couple map, which is also a collection of R -linear maps. For such couples, $\langle A, B, f, g, h \rangle$, we assume f, g , and h have bidegrees $(+1, -1)$, $(0, 0)$, $(-1, 0)$.

There is a homotopy theory in $\mathcal{E}\mathcal{C}_R$ given by

DEFINITION 1. If $\langle A, B, f, g, h \rangle \in \mathcal{E}\mathcal{C}_R$, set

$$\begin{aligned} \overline{A}^{p,q} &= A^{p,q} \oplus B^{p,q} & \tilde{f}(x, y) &= (f(x), 0), \\ \overline{B}^{p,q} &= B^{p,q} \oplus B^{p,q} \oplus B^{p-1,q}, & \tilde{g}(x, y) &= (g(x) - y, y, -gh(y)), \\ & & \tilde{h}(x, y, z) &= (h(x + y), z + gh(y)). \end{aligned}$$

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