GLOBAL CONTINUOUS SOLUTIONS OF HYPERBOLIC SYSTEMS OF QUASI-LINEAR EQUATIONS[1]

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Recently there have appeared a number of results on global solutions of the Cauchy problem for hyperbolic systems of quasi-linear equations [2], [3], [4], [5]. These solutions are in general discontinuous. In certain cases, however, such as the interaction of two rarefaction waves in gas dynamics, it is known that the Cauchy problem has a global continuous solution [1, pp. 191–197]. In this announcement we outline a proof that a global continuous solution exists and is unique for a two-dimensional system provided the Riemann invariants associated with the initial data satisfy certain monotonicity and continuity conditions.

Let $\lambda^+(r, s)$, $\lambda^-(r, s)$ be C^1 real-valued functions on a domain $D \subset R_2$, with

$$(1) \qquad \lambda^+(r, s) > \lambda^-(r, s), \qquad \partial\lambda^+(r, s)/\partial r > 0, \qquad \partial\lambda^-(r, s)/\partial s > 0$$

for $(r, s) \in D$. Consider the two-dimensional system of quasi-linear equations in Riemann invariant form

$$(2) \qquad r_t + \lambda^+(r, s) r_x = 0, \qquad s_t + \lambda^-(r, s) s_x = 0$$

where $r(t, x)$ and $s(t, x)$ are real-valued functions of two scalar variables. We seek a solution of the Cauchy problem in the halfplane $\{(t, x) \in R_2 : t \geqq 0\}$ with initial conditions

$$(3) \qquad r(0, x) = r^0(x), \qquad s(0, x) = s^0(x), \qquad -\infty < x < +\infty.$$

Let $G_T = \{(t, x) \in R_2 : 0 \leqq t < T\}$ for $0 < T \leqq +\infty$. A pair of Lipschitz continuous functions $(r(t, x), s(t, x))$, $(t, x) \in G_T$, is called a Lipschitz continuous solution of the Cauchy problem (2), (3) if $r(t, x)$ is constant on the integral curves

$$(4) \qquad x'(t) = \lambda^+(r(t, x), s(t, x)),$$

$s(t, x)$ is constant on the integral curves

$$(5) \qquad x'(t) = \lambda^-(r(t, x), s(t, x)),$$

[1] These results are part of the author's Ph.D. thesis written under Joel A. Smoller at the University of Michigan.