SUBDIRECT PRODUCTS OF SEMIGROUPS AND RECTANGULAR BANDS

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Let $\{S_\alpha : \alpha \in A\}$ be a family of semigroups. If p_α is the natural projection from $\Pi\{S_\alpha : \alpha \in A\}$ onto S_α, then a subsemigroup D of $\Pi\{S_\alpha : \alpha \in A\}$ is called a subdirect product of $\{S_\alpha : \alpha \in A\}$ if $p_\alpha(D) = S_\alpha$ for all $\alpha \in A$.

If L and R are sets then the semigroup $B = L \times R$ with $(\lambda_1, \rho_1) \cdot (\lambda_2, \rho_2) = (\lambda_1, \rho_2)$ is called a rectangular band. Our main result, Theorem 1, determines all subdirect products of a semigroup S and a rectangular band B. Elements of $S \times B$ will be denoted by $(s; \lambda, \rho)$ $(s \in S, \lambda \in L, \rho \in R)$.

Proofs of the following results will appear elsewhere. See [1] for all undefined concepts.

THEOREM 1. *Let S be a semigroup and $B = L \times R$ be a rectangular band. If $\mathfrak{L}$ is the set of all left ideals of S and $\mathfrak{R}$ is the set of all right ideals of S, then two mappings ϕ: $L \to \mathfrak{R}$ and ψ: $R \to \mathfrak{L}$ satisfying*

$$S = \bigcup\{\phi(\lambda) : \lambda \in L\} = \bigcup\{\psi(\rho) : \rho \in R\}$$

determine a subdirect product $D \subseteq S \times B$ by

$$D = \bigcup\{D(\lambda, \rho) : (\lambda, \rho) \in B\},$$

where

$$D(\lambda, \rho) = \{(x; \lambda, \rho) : x \in \phi(\lambda) \cap \psi(\rho)\}.$$

Moreover, the correspondence $(\phi, \psi) \to D$ is one-to-one onto the set of all subdirect products of S and B.

One application of this theorem is

COROLLARY 1. *Let S be a semigroup and $B = L \times R$ be a rectangular band. The only subdirect product of S and B is the direct product of S and B if and only if one of the following is satisfied:*

(i) *S is right simple, and B is a left zero semigroup, $B \cong L$,*

(ii) *S is left simple, and B is a right zero semigroup, $B \cong R$,*

(iii) *S is a group, or*

(iv) *B is trivial, $|B| = 1$.*

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