EXISTENCE AND REGULARITY OF SOLUTIONS TO ELLIPTIC CALCULUS OF VARIATIONS PROBLEMS AMONG SURFACES OF VARYING TOPOLOGICAL TYPE AND SINGULARITY STRUCTURE

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DEFINITIONS AND NOTATION. (1) m and n denote positive integers.

(2) H^k *denotes Hausdorff* k *dimensional measure in* R^{m+n} *for* $k=m,\ m-1$.

(3) G_m^{m+n} denotes the Grassmann manifold of unoriented m plane directions in R^{m+n} (which can be regarded as the space of all unoriented m planes through the origin in R^{m+n}).

(4) A $C^{(k)}$ *integrand* [*real analytic integrand*] is a function [real analytic function] F: $G_m^{m+n} \to R \cap \{t: t>0\}$ whose partial derivatives up to order k exist and are continuous. Here k denotes either a positive integer or ∞.[2]

(5) A *surface* S is a compact m-rectifiable subset of R^{m+n}. If S is a surface, then, for H^m almost all $x \in S$, S has an approximate tangent m plane direction at x, denoted $\mathrm{S}(x)$.

(6) The integral of an integrand F over a surface S is defined to be

$$F(S) = \int_{x \in S} F(\mathrm{S}(x))\, dH^m x.$$

(7) A *boundary* B is a compact $(m-1)$-rectifiable subset of R^{m+n} with $H^{m-1}(B) < \infty$.

(8) $\mathbf{G}$ denotes the category of all finitely generated abelian groups. If B is a boundary, S is a surface, and $G \in \mathbf{G}$, we denote by $H_{m-1}(B; G)$ and $H_{m-1}(B \cup S; G)$ the $m-1$ dimensional Vietoris homology groups of B and $B \cup S$, respectively, with coefficients in G. If $\sigma \in H_{m-1}(B; G)$ we say that S *spans* σ if and only if $i_*(\sigma) = 0$ where

$$i_*: H_{m-1}(B; G) \to H_{m-1}(B \cup S; G)$$

is induced by the inclusion i: $B \to B \cup S$.

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[2] The existence and regularity results of this paper have recently been extended to apply to integrands F: $R^{m+n} \times G_m^{m+n} \to R$ which are elliptic on each tangent space. For such integrands one sets $F(S) = \int F(x, S(x))\, d(H^m S)x$.