ESSENTIAL SPECTRA OF ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS[1]

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Let A be a closed, densely defined operator in a Banach space X. There are several definitions of the "essential" spectrum of A (cf. [1], [2]). According to Wolf [3], [4] it is the complement in the complex plane of the Φ-set of A. The Φ-set Φ_A of A is the set of points λ for which

(a) $\alpha(A-\lambda)$, the dimension of the null space of $A-\lambda$, is finite

(b) $R(A-\lambda)$, the range of $A-\lambda$, is closed

(c) $\beta(A-\lambda)$, the codimension of $\mathrm{R}(A-\lambda)$, is finite.

We denote the essential spectrum according to this definition by $\sigma_{ew}(A)$. The set $\sigma_{em}(A)$, as defined in [1], [2] is obtained by adding to $\sigma_{ew}(A)$ those points λ for which $\alpha(A-\lambda)\neq\beta(A-\lambda)$. It is the largest subset of $\sigma(A)$ which remains invariant under compact perturbations. Finally, to obtain the set $\sigma_{eb}(A)$, which is the essential spectrum according to Browder [5], we add to $\sigma_{em}(A)$ those points of $\sigma(A)$ which are not isolated.

Interest in the sets $\sigma_{ew}(A)$, $\sigma_{em}(A)$, $\sigma_{eb}(A)$ is centered about the fact that they remain invariant under certain perturbations of A. In particular one has

THEOREM 1. *Let A and B be closed densely defined operators in X. If $\lambda_0\in\rho(A)\cap\rho(B)$ and $(A-\lambda_0)^{-1}-(B-\lambda_0)^{-1}$ is a compact operator in X, then*

$$\sigma_{ew}(A) = \sigma_{ew}(B) \tag{1}$$

and

$$\sigma_{em}(A) = \sigma_{em}(B). \tag{2}$$

Moreover, if the complement $C\sigma_{em}(A)$ of $\sigma_{em}(A)$ is connected, then

$$\sigma_{eb}(A) = \sigma_{eb}(B). \tag{3}$$

This theorem was proved in [2] under the additional assumption that $D(B)\supseteq D(A)$. For selfadjoint operators the basic idea was employed by Birman [6], Wolf [4] and Rejto [7].

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