THE CAUCHY INTEGRAL FOR DIFFERENTIAL FORMS

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In a previous note, we introduced a certain determinant of differential forms (cf. formula (1.3) of [3]), which led to an elementary proof of the Cauchy integral formula for holomorphic functions of several complex variables. We now propose to amplify this procedure in order to obtain some integral representations for exterior differential forms.

We shall be considering certain mappings $\psi(\zeta, z)$ and $f(\zeta, z)$ of an open set $V\subset \boldsymbol{C}^n\times \boldsymbol{C}^n$ into $\boldsymbol{C}^n$, where ψ is of class C^∞ and f is holomorphic. Writing $\psi=(\psi_1, \cdots, \psi_n)$ and $f=(f_1, \cdots, f_n)$, we set $\langle\psi, f\rangle=\psi_1 f_1+ \cdots +\psi_n f_n$. Henceforth, we shall always assume that $\langle\psi, f\rangle\neq 0$ at the points under consideration. Now, instead of merely considering a single smooth mapping ψ, we take n such mappings, $\psi^{(1)}, \psi^{(2)}, \cdots, \psi^{(n)}$. Each of these will be regarded as a column in terms of its components. We shall, furthermore, use the vector-valued differential forms $\bar\partial_\zeta\psi=\sum_{j=1}^n \psi_{\bar\zeta_j}d\bar\zeta_j$ and $\bar\partial_z\psi=\sum_{j=1}^n\psi_{\bar z_j}d\bar z_j$. With this notation, we look at the $n\times n$ determinant

$$(1)\qquad D(\langle\psi^{(1)},f\rangle^{-1}\psi^{(1)}, \underset{j=2,\cdots,\mu}{\bar\partial_z(\langle\psi^{(j)},f\rangle^{-1}\psi^{(j)})}, \underset{k=\mu+1,\cdots,n}{\bar\partial_\zeta(\langle\psi^{(k)},f\rangle^{-1}\psi^{(k)})}),$$

which will be viewed as a double form on V (cf. [4, p. 35]). As in [3], we can now state

PROPOSITION 1. *The expression* (1) *is independent of the choice of the mapping* $\psi^{(1)}$.

Along the same direction, we also have

PROPOSITION 2.

$$\text{(i)}\qquad D(\bar\partial_z(\langle\psi^{(1)},f\rangle^{-1}\psi^{(1)}), \underset{j=2,\cdots,\mu}{\bar\partial_z(\langle\psi^{(j)},f\rangle^{-1}\psi^{(j)})}, \underset{k=\mu+1,\cdots,n}{\bar\partial_\zeta(\langle\psi^{(k)},f\rangle^{-1}\psi^{(k)})}) = 0;$$

$$\text{(ii)}\qquad D(\bar\partial_\zeta(\langle\psi^{(1)},f\rangle^{-1}\psi^{(1)}), \underset{j=2,\cdots,\mu}{\bar\partial_z(\langle\psi^{(j)},f\rangle^{-1}\psi^{(j)})}, \underset{k=\mu+1,\cdots,n}{\bar\partial_\zeta(\langle\psi^{(k)},f\rangle^{-1}\psi^{(k)})}) = 0.$$

With the aid of Propositions 1 and 2, we can derive

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