ON THE STRUCTURE OF THE SET OF SUBSEQUENTIAL LIMIT POINTS OF SUCCESSIVE APPROXIMATIONS

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1. **Introduction.** Let $A: S \rightarrow S$ be continuous, where S is a nonempty metric space. Much current research is concerned with the existence of fixed points of A. Given $x \in S$, it is natural to ask if the (Picard) sequence of iterates $\{A^m(x)\}_{m=1}^{\infty}$ converges to a fixed point of A. In certain circumstances, answers to questions of this general type have been given by Edelstein [1], [2] and Browder and Petryshyn [3], [4]. It appears to be of interest to consider first the structure of the set of subsequential limit points of the sequence of iterates $\{A^m(x)\}_{m=1}^{\infty}$, with the ultimate end in view of ascertaining when this sequence actually converges. This is the general approach followed in this announcement.

2. **Main results.** It is quite popular to consider a function A which is a contraction over its entire domain of definition. The following theorem makes use of a contractive hypothesis, but only with respect to the set of fixed points of A (see hypothesis (ii) below).

In the sequel, $F(A)$ will denote the set of fixed points of A, that is, $F(A) = \{x \in S \mid A(x) = x\}$. Also, $d(x, F(A))$ will denote the distance between the point x and the set $F(A)$, that is, $d(x, F(A)) = \inf_{p \in F(A)} d(x, p)$. Further, for $x \in S$, $\mathcal{L}(x)$ will denote the set of subsequential limit points of the sequence of iterates $\{A^m(x)\}_{m=1}^{\infty}$. As usual, a subset K of S will be said to be compact if every sequence of points from K contains a subsequence which converges to a point in S.

THEOREM 1. *Suppose*
(i) *$F(A)$ is nonempty and compact;*
(ii) *for each $x \in S$, with $x \notin F(A)$, one has*

$$d(A(x), F(A)) < d(x, F(A)).$$

Then, for $x \in S$, the set $\mathcal{L}(x)$ is a closed and connected subset of $F(A)$. Either $\mathcal{L}(x)$ is empty, or it contains exactly one point, or it contains

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