THE CONTRACTIBILITY OF CERTAIN TEICHMÜLLER SPACES[1]

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1. **Introduction.** From the work of Fenchel and Nielsen [7] or from recent work of Keen [8], it is known that for every finitely generated Fuchsian group Γ of the first kind the Teichmüller space $T(\Gamma)$ is contractible. We extend this result here, proving that the Teichmüller space of every nonelementary finitely generated Fuchsian group is contractible. We give only an indication of the proof. The complete proof, which is rather technical, will appear later.

2. **Definitions.** Let $U=\{z\in \mathbf{C}\colon \operatorname{Im} z>0\}$ be the upper half plane. Let Γ be a finitely generated group of Möbius transformations which map U onto itself. The *limit set* $L(\Gamma)$ consists of all points in the extended plane which are limit points of some orbit Γz, $z\in \mathbf{C}$. We assume that Γ is *Fuchsian*, meaning that $L(\Gamma)$ is a subset of the extended real axis, and *nonelementary*, meaning that $L(\Gamma)$ contains at least three points. We shall in fact require that $0, 1, \infty \in L(\Gamma)$. This involves no loss of generality, for if Γ is any nonelementary Fuchsian group, there is a Möbius transformation $A\colon U\to U$ such that the Fuchsian group $A\circ\Gamma\circ A^{-1}$ meets our requirement, and the Teichmüller spaces $T(\Gamma)$ and $T(A\circ\Gamma\circ A^{-1})$ are homeomorphic [1, Theorem 8].

Let $L^\infty(\Gamma)$ be the Banach subspace of $L^\infty(U, \mathbf{C})$ consisting of the functions $\mu(z)$ which satisfy

$$\mu(Az)A'(z)^*/A'(z) = \mu(z) \qquad \text{for all } A \text{ in } \Gamma.$$

The open unit ball in $L^\infty(\Gamma)$, denoted by $M(\Gamma)$, is called the set of *Beltrami differentials*. Each Beltrami differential μ determines a quasiconformal map w_μ of U onto itself which leaves the points $0, 1, \infty$ fixed and satisfies in U the Beltrami equation $w_{\bar z}=\mu w_z$.

We say that μ and ν in $M(\Gamma)$ are *strongly equivalent* if $w_\mu(x)=w_\nu(x)$ for all real x. The set of strong equivalence classes is the *Teichmüller space* $T(\Gamma)$. Let $\Phi\colon M(\Gamma)\to T(\Gamma)$ be the map which sends each μ in $M(\Gamma)$ to its strong equivalence class. We give $T(\Gamma)$ the quotient topology induced by Φ.

If $w_\mu(x)=w_\nu(x)$ for all x in $L(\Gamma)$ we say that μ and ν in $M(\Gamma)$ are

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