THE DIRICHLET PROBLEM FOR NONUNIFORMLY ELLIPTIC EQUATIONS[1]

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Introduction. Let Ω be a bounded domain in E^n. The operator

$$Qu = a^{ij}(x, u, u_x)u_{x_ix_j} + a(x, u, u_x)$$

acting on functions $u(x) \in C^2(\Omega)$ is *elliptic* in Ω if the minimum eigenvalue $\lambda(x, u, p)$ of the matrix $[a^{ij}(x, u, p)]$ is positive in $\Omega \times E^{n+1}$. Here

$$u_x = (u_{x_1}, \cdots u_{x_n}), \qquad p = (p_1, \cdots p_n)$$

and repeated indices indicate summation from 1 to n. The functions $a^{ij}(x, u, p)$, $a(x, u, p)$ are defined in $\Omega \times E^{n+1}$. If furthermore for any $M > 0$, the ratio of the maximum to minimum eigenvalues of $[a^{ij}(x, u, p)]$ is bounded in $\Omega \times (-M, M) \times E^n$, Qu is called *uniformly elliptic*. A solution of the *Dirichlet problem* $Qu = 0$, $u = \phi(x)$ on $\partial\Omega$ is a $C^0(\bar{\Omega}) \cap C^2(\Omega)$ function $u(x)$ satisfying $Qu = 0$ in Ω and agreeing with $\phi(x)$ on $\partial\Omega$.

When Qu is elliptic, but not necessarily uniformly elliptic, it is referred to as *nonuniformly elliptic*. In this case it is well known from two dimensional considerations, that in addition to smoothness of the boundary data $\partial\Omega$, $\phi(x)$ and growth restrictions on the coefficients of Qu, geometric conditions on $\partial\Omega$ may play a role in the solvability of the Dirichlet problem. A striking example of this in higher dimensions is the recent work of Jenkins and Serrin [**4**] on the minimal surface equation, mentioned below.

The Dirichlet problem for general classes of nonuniformly elliptic equations has been considered by Gilbarg [**1**], Stampacchia [**7**], Hartman and Stampacchia [**2**], Hartman [**3**], and Motteler [**6**]. We announce below some theorems which extend the results of these authors. The detailed proofs will appear elsewhere.

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Equations of the form $a^{ij}(u_x)u_{x_ix_j} = 0$. Prior to stating our theorem we formulate a generalization of the well-known bounded slope condition, or B.S.C., used in [**2**], [**3**], and [**7**]. Let Γ be the $n-1$ dimen-

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