HOLOMORPHIC APPROXIMATION ON REAL SUBMANIFOLDS OF A COMPLEX MANIFOLD[1]

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Let X be a complex manifold, let $\mathcal{O}$ be the sheaf of germs of holomorphic functions on X, and let $\Gamma(K, \mathcal{O})$ denote the ring of sections over $K \subset X$ (holomorphic functions on K, see [5] as a general reference for the terminology used here). If K is a compact set in X, let $C(K)$ be the Banach algebra of continuous complex-valued functions on K with respect to the maximum norm on K. Let $A(K)$ denote the closure of $\Gamma(K, \mathcal{O})$ in $C(K)$, and if $U \supset K$, let $A(K, U)$ denote the closure in $C(K)$ of the restriction of $\Gamma(U, \mathcal{O})$ to K.

It is an old problem to try to determine under what conditions $A(K) = C(K)$ or $A(K, U) = C(K)$, for some U. For instance, if $X = \mathbf{C}^n = U$, then $A(K, \mathbf{C}^n)$ is the polynomial algebra important in the theory of polynomial approximation, and sets for which $A(K, \mathbf{C}^n) = C(K)$ are necessarily polynomially convex. Also one is interested in the nature of the spectrum of $A(K)$ or $A(K, U)$ which can be interpreted as a holomorphic hull of some type, depending on what U is. This leads to the study of holomorphic or polynomial convexity of sets (the property of being equal to the spectrum or maximal ideal space of the algebra) and approximation theorems always give information about the relation of the various spectra involved (see [1], [11], [12]). In this note we announce certain results on approximation of continuous functions, in the case where K is a compact real submanifold of X (differentiable), with or without boundary.

Let M be a real submanifold (C^∞) of X. Let $T(X)$ be the tangent bundle to X, and let $J: T(X) \to T(X)$ be the canonical involution ($J^2 = -I$) given by the complex structure on X. Let $T(M)$ be the tangent bundle to M, which is, in a natural way, a real subbundle of $T(X) \mid M$. Set

$$H_p(M) = JT_p(M) \cap T_p(M), \qquad p \in M.$$

Then $H_p(M)$ is a complex subspace of $T_p(X)$, and is called the *holomorphic tangent space to M at p*. We will denote by $m_p(M)$ the complex dimension of $H_p(M)$.

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