THE CAUCHY INTEGRAL FOR FUNCTIONS OF SEVERAL COMPLEX VARIABLES

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This note contains a new contribution to the already vast literature on the topic under discussion. While we were led to the present considerations through a remark in a paper of Bochner [4] and originally arrived at our results more or less independently, we found, upon turning to the literature of the intervening twenty years, that our work was quite closely related to papers by Weil, Sommer, Arens, Leray, Norguet, Waelbroeck, Gleason, and Aĭzenberg [11], [9], [3], [6], [7], [10], [5], [1]. Nevertheless, our approach offers some clarification of the existing theory and, indeed, we demonstrate this assertion by discussing a few applications. The subjects outlined here constitute a portion of material that will appear in forthcoming publications.

1. Cauchy-Fantappiè forms. Let $\zeta \in C^n$. We shall be interested in certain mappings of neighborhoods of ζ into C^n. Let us, in particular, consider two such mappings, ψ and f, where ψ is of class C^∞ and f is holomorphic. Writing $\psi = (\psi_1, \cdots, \psi_n)$ and $f = (f_1, \cdots, f_n)$, we set $\langle \psi, f\rangle = \psi_1 f_1 + \cdots + \psi_n f_n$. As usual, we decompose the operation d of exterior differentiation into its two components ∂, $\bar\partial$, of bidegree (1, 0) and (0, 1) respectively, so that $d = \partial + \bar\partial$. Next, we introduce the two differential forms $\omega(f) = df_1 \wedge \cdots \wedge df_n$ and

$$(1.1)\quad \omega'(\psi) = \sum_{j=1}^{n} (-1)^{j-1}\psi_j \bar\partial\psi_1 \wedge \cdots \wedge \bar\partial\psi_{j-1} \wedge \bar\partial\psi_{j+1} \wedge \cdots \wedge \bar\partial\psi_n.$$

If $\langle \psi, f\rangle\,(\zeta) \neq 0$, then the differential form

$$(1.2)\qquad (-1)^{n(n-1)/2}(n-1)!(2\pi i\langle\psi, f\rangle)^{-n}\omega'(\psi) \wedge \omega(f)$$

will be called a Cauchy-Fantappiè (C-F) form of type f at ζ. When $\psi = \bar f$, we shall speak of the Bochner-Martinelli (B-M) form of type f. The fundamental structure of C-F forms follows from

THEOREM 1.1. (i) $\bar\partial\langle\psi, f\rangle^{-n}\omega'(\psi) = 0$.

(ii) *If the functions* $\langle\psi^{(j)}, f\rangle$, $j = 1, 2$, *do not vanish on an open neighborhood* V_ζ *of* ζ, *then there exists a differential form* β *of bidegree* $(0, n-2)$ *on* V_ζ, *such that*

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