EXISTENCE AND PERTURBATION THEOREMS FOR NONLINEAR MAXIMAL MONOTONE OPERATORS IN BANACH SPACES

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Let X be a Banach space, X^* its conjugate space with the pairing between w in X^* and u in X denoted by (w, u). If T is a mapping (in general, nonlinear) with domain $D(T)$ in X and range $R(T)$ in X^*, T is said to be monotone if for all u and v of $D(T)$

$$(T(u) - T(v), u - v) \geqq 0. \tag{1}$$

More generally, a subset G of $X \times X^*$ is said to be a monotone set if for each pair $[u_1, w_1]$ and $[u_2, w_2]$ in G, we have

$$(w_2 - w_1, u_2 - u_1) \geqq 0. \tag{2}$$

Such a set G is said to be maximal monotone if it is maximal among monotone sets in the sense of set inclusion, and a mapping T is said to be maximal monotone if its graph $G(T)$ is a maximal monotone set.

For reflexive Banach spaces X and mappings T with $D(T)=X$, the basic result obtained independently by Browder [2] and Minty [20] states that if T is a monotone operator from $D(T)=X$ to X^* which is *hemicontinuous*, (i.e. continuous from each line segment in $D(T)$ to the weak topology on X^*), and *coercive*, i.e.

$$(Tu, u)/\|u\| \to +\infty \quad (\text{as } \|u\| \to +\infty) \tag{3}$$

then the range $R(T)$ of T is the whole of the space X^*. This theorem and its extensions to various classes of operators T from all of X to X^* which satisfy modified monotonicity conditions (Browder [4], [9], [10], [11], [12], Leray-Lions [15], Hartman-Stampacchia [14]) are the basis of the application of the theory of monotone operators to obtain general existence theorems for nonlinear elliptic boundary value problems.

For nonelliptic problems, the corresponding reduction to equations for nonlinear operators acting from a reflexive Banach space X to its dual space X^* yields operators T which are only defined on a dense subset $D(T)$ of X. Results covering the principal parabolic and hyperbolic problems were obtained in Browder [3], [5], [7] for operators T of the form $T=L+T_0$, where T_0 is an everywhere defined hemicontinuous monotone nonlinear operator from X to X^* which is coercive and maps bounded subsets of X into bounded sub-