DEVELOPMENTS IN THE CLASSICAL NEVANLINNA THEORY OF MEROMORPHIC FUNCTIONS[1,2]

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1. Nevanlinna's theory of meromorphic functions is about forty years old. The ideas of this theory reveal a "fine structure" of the distribution of values that was not visible to the older investigations of Picard, Borel and others. The aim of this paper is to report on some results concerning this "fine structure," especially on those based on the notion of Nevanlinna deficiency. It covers, therefore, only a very small part of the work on Nevanlinna Theory. In particular the many important generalizations of the classical Nevanlinna theory are not treated at all (Algebroid functions: H. Selberg [**63**], [**64**]; Meromorphic curves: L. Ahlfors [**1**], H. and J. Weyl [**G**]; Mappings of a Riemann surface into another Riemann surface: S. S. Chern [**7**], L. Sario [**60**], [**61**] and (with K. Noshiro) [**F**]; Holomorphic mappings of complex analytic manifolds: R. Bott and S. S. Chern [**6**]).

2. By a meromorphic function I shall mean a function meromorphic in $|z| < \infty$. The symbol $f(z)$ will always denote a meromorphic function. The core of Nevanlinna's Theory is expressed in the two "Fundamental Theorems." Some notation is needed for their statement. Let $n(r, f)$ denote the number of poles of $f(z)$ in $|z| \leqq r$, each pole counted with its proper multiplicity (simple pole once, double pole twice, $\cdots$). Then the number of solutions of $f(z) = c$ in $|z| \leqq r$ is given by $n(r, 1/(f-c))$. Let

$$N(r, \infty) = N(r,f) = \int_0^r (n(t,f) - n(0,f))t^{-1}\,dt + n(0,f)\log r,$$

$$m(r, \infty) = m(r,f) = (1/2\pi)\int_0^{2\pi} \log^+ |f(re^{i\theta})|\,d\theta$$

$$\cdot(\log^+|n| = \max\{\log|n|, 0\}).$$

$$N(r, c) = N(r, 1/(f-c)), \qquad m(r, c) = m(r, 1/(f-c)) \quad (c \leqq \infty).$$

$N(r, c)$ is a smoothed counting function of the c-points of $f(z)$, $m(r, c)$ is a "proximity function" measuring how close $f(z)$ comes to c, on the average, on $|z| = r$. We can now state the

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