THE HITTING CHARACTERISTICS OF A STRONG MARKOV PROCESS, WITH APPLICATIONS TO CONTINUOUS MARTINGALES IN R^n

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1. **Introduction.** M. Arbib showed in [1] that, essentially, on the real line a continuous path process with the same "hitting characteristics" as a diffusion was itself a diffusion (strong Markov process). His methods did not lend themselves to more general processes. The purpose of this note is to give a general characterization along this line, for right continuous, nonterminating, quasi-left continuous strong Markov processes with left limits, taking their values in a locally compact, noncompact second countable space E. We also give some interesting consequences concerning continuous martingales in R^n. Full proofs of these and related results will appear elsewhere.

2. **Hitting characteristics.** Let $\hat{X}$ be a process as above, described by measures $\hat{P}^x$ $(x \in E)$ on the space of paths (assume that the function $x \to \hat{P}^x(A)$ is Borel measurable for all Borel $A \subset E$, and that $\hat{P}^x$ $(X_0 = x) = 1)$. Let $\boldsymbol{F}_t$ be the σ-field generated by the path functions X_s $(s \leqq t)$, and let $\boldsymbol{F}_R^+$ be the σ-field of the stopping time R $(A \in \boldsymbol{F}_R^+$ if $A \cap \{R < t\} \in \boldsymbol{F}_t$ for all $t)$. Another process X, described by a measure P on the same path space, will be said to have the same *hitting characteristics* as $\hat{X}$ if

$$\text{(H1)} \qquad E[T_{\bar{G}^c} \circ \theta_R \mid \boldsymbol{F}_R^+] = \hat{E}^{X_R}[T_{\bar{G}^c}] \quad \text{P-a.s.,}$$

$$\text{(H2)} \qquad E[I_B \circ X_{T_{\bar{G}^c}} \circ \theta_R \mid \boldsymbol{F}_R^+] = E^{X_R}[I_B \circ X_{T_{\bar{G}^c}}] \quad \text{P-a.s.}$$

for every stopping time R, Borel set $B \subset E$, and open set $G \subset E$ with compact closure ($T_{\bar{G}_n^c}$ is the first hitting time of the complement of the closure of G, and θ_R is the shift by R).

We write μ for the distribution of X_0 under P.

THEOREM 1. *If X and $\hat{X}$ have the same hitting characteristics as described above, and if there is a sequence of sets $G_n \nearrow E$, G_n open with compact closure, such that $x \to \hat{E}^X[T_{\bar{G}_n^c}]$ is a bounded function on E, then $P = \hat{P}^\mu$—i.e., X is a strong Markovprocess.*

The existence of the sets G_n follows whenever $\hat{X}$ is, say, a Feller