ON VON KARMAN'S EQUATIONS AND THE BUCKLING OF A THIN ELASTIC PLATE

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The object of this note is to demonstrate the applicability of the methods of nonlinear functional analysis in the investigation of a complex physical problem. In 1910 T. von Karman [9] introduced a system of 2 fourth order elliptic quasilinear partial differential equations which can be used to describe the large deflections and stresses produced in a thin elastic plate subjected to compressive forces along its edge. The most interesting phenomenon associated with this nonlinear situation is the appearance of "buckling," i.e. the plate may deflect out of its plane when these forces reach a certain magnitude. Mathematically this circumstance is expressed by the multiplicity of solutions of the boundary value problem associated with von Karman's equations. With the aid of the modern theory of linear elliptic partial differential equations together with functional analysis on a suitably chosen Hilbert function space, *we are able to use the structural pattern of the nonlinearity implicit in Karman's equations* to obtain a qualitative nonuniqueness theory for this problem.

Among the previous studies of buckling of plates are those of Friedrichs and Stoker [5] and Keller, Keller and Reiss [6], who study only radially symmetric solutions of circular plates. Numerical studies for rectangular plates have been given by Bauer and Reiss [2] among others. Karman's equations for general domains have been studied by Fife [4] and Morosov [8] in other connections. The authors are grateful to Professors S. Agmon and W. Littman for helpful suggestions. This research was partially supported by the National Science Foundation Grant No. GP-3904 and the Air Force Office of Scientific Research Grant No. 883-65.

1. **Classical and generalized solutions (for a clamped plate).** Let Ω be a bounded domain in R^2 with boundary $\partial\Omega$ consisting of a finite number of arcs on each of which a tangent rotates continuously. Defined over Ω, we consider the following system of partial differential equations and boundary conditions:

$$(1)\qquad \begin{aligned} \Delta^2 f &= -[w, w],\\ \Delta^2 w &= \lambda[F, w] + [f, w], \end{aligned}$$

$$(2)\qquad w = w_x = w_y = f = f_x = f_y = 0 \quad \text{on} \quad \partial\Omega$$