ROYDEN'S MAP BETWEEN RIEMANN SURFACES[1]

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Let R and R_j ($j=1, 2$) be Riemann surfaces, either open or closed. We denote by $M(R)$ *Royden's algebra* associated with R, and by R^* *Royden's compactification* of R (see [5], [6], and [7]). We have seen in [5] that every algebraic isomorphism of $M(R_1)$ onto $M(R_2)$ induces (and is induced by) a quasiconformal mapping of R_1 onto R_2. In other words, the algebraic structure of $M(R)$ characterizes the quasiconformal structure of R. In this connection there naturally arises the following question: What can we say about the topological structure of R^*? This question leads us to a new notion, Royden's map, which seems to be of considerable function-theoretic interest.

Here we report, without proofs, some of the properties of Royden's maps. Details will be published elsewhere.

1. **Moduli of A-sets.** An open subset G of R is called *normal* if for any point z in ∂G there exists a parametric disk U, with center z, such that $\partial G \cap U$ is a simple arc connecting two boundary points of U.

An *A-set* A is a pair (G_1, G_2) of two nonempty normal open sets G_1 and G_2 in R with $G_1 \supset \overline{G}_2$. An annulus in a parametric disk is an example of an A-set.

We associate with an A-set $A=(G_1, G_2)$ a family $\{\phi\}$ of functions ϕ which are continuous on $\overline{G}_1 - G_2$, of class C^1 in $G_1-\overline{G}_2$, and have boundary values $\phi | \partial G_j = j$ ($j=1, 2$). The *modulus* of A, denoted mod A, is the number in $[0, \infty)$ given by

$$\text{mod } A = 2\pi / \inf_{\phi \in \{\phi\}} D(\phi), \tag{1}$$

where $D(\phi)$ is the Dirichlet integral of ϕ taken over $G_1 - \overline{G}_2$. If A is an annulus in a parametric disk, then this definition coincides with the usual one.

2. **Royden's map.** A topological mapping T of R_1 onto R_2 carries an A-set $A=(G_1, G_2)$ on R_1 to the A-set $TA=(TG_1, TG_2)$ on R_2. We call T a *Royden's map* if there exists a constant $K(A) \geqq 1$ such that

$$K(A)^{-1} \text{ mod } A \leqq \text{mod } TA \leqq K(A) \text{ mod } A, \tag{2}$$

for every A-set A on R_1. Here $K(A)$ may depend on A. If we can find

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