CENTRAL IDEMPOTENTS IN GROUP ALGEBRAS

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1. **Introduction.** The group algebra $L^1(G)$ of a group G consists of all complex functions f on G which have finite norm:

$$\|f\| = \sum_{x \in G} |f(x)| < \infty.$$

Multiplication is given by convolution:

$$f * g(x) = \sum_{y \in G} f(xy^{-1})g(y).$$

A function $f \in L^1(G)$ is an *idempotent* if $f * f = f$; it is *central* if $f(xy) = f(yx)$ for all $x, y \in G$. The *support* of f is the set of x for which $f(x) \neq 0$. The *support group* of f is the subgroup generated by the support of f.

The idempotents on abelian groups have been completely characterized [**1**], [**2**]; in particular they have finite support groups. Rudin [**3**] gives examples of noncentral idempotents on nonabelian groups which have infinite support and, *a fortiori*, infinite support groups.

The purpose of this paper is to answer affirmatively the question raised [**3**], [**4**] as to whether or not central idempotents have finite support groups.

THEOREM. *The support group of a central idempotent is finite.*

2. **Proof of the theorem.** Let f be a central idempotent on G. We can assume that G is the support group of f.

If G' is the commutator subgroup of G then it follows [**4**, Theorem 2.2] that G' has finite index in G. We will show that $Z =$ the center of G also has finite index in G. But this implies that G' is finite (see, for example, [**5**, Theorem 15.1.13]) so that G must also be finite.

Let $S = \{x_1, x_2, \cdots\}$ be the support of f. Since f is central and has finite norm, $\{gx_ig^{-1} : g \in G\}$ is finite for each x_i so that each x_i commutes with the elements of a subgroup of finite index. Let H_n be the normal subgroup generated by $x_1, x_2, \cdots, x_n$ and let Z_n be the ele-

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