# THE NOETHERIAN DIFFERENT OF PROJECTIVE ORDERS[1]

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1. Let $K$ be a commutative ring and $\Lambda$ a $K$-algebra (all rings will have identities and all modules will be unitary). Let $Z(\Lambda)$ be the center of $\Lambda$. Let $\phi_\Lambda: \Lambda \otimes_K \Lambda^0 \to \Lambda$ be given by $\phi_\Lambda(x \otimes y^0) = xy$. This is a homomorphism of left $\Lambda^e$-modules ($\Lambda^e = \Lambda \otimes_K \Lambda^0$). The set

$$N(\Lambda/K) = \{a \in Z(\Lambda) : \exists f \in \operatorname{Hom}_{\Lambda^e}(\Lambda, \Lambda^e) \text{ with } \phi_\Lambda f = aI_\Lambda\}.$$

is an ideal in $Z(\Lambda)$ called the *Noetherian different* of $\Lambda$ over $K$.

In this note we announce an extension of a result of D. G. Higman [2] to projective central orders over integrally closed integral domains. Details will appear in a paper of the same title in the *Journal für die reine und angewandte Mathematik* (*Crelle*).

It should be noted here that several authors have studied this ideal. See, for example, the papers listed in the bibliography.

2. Let $K$ be an integrally closed integral domain with quotient field $L$. Let $\Lambda$ be a $K$-order in a central simple $L$-algebra, $\Sigma$, which is projective as a $K$-module. Let $T: \Sigma \to L$ be the reduced trace from $\Sigma$ to $L$. The hypothesis that $K$ is integrally closed in $L$ and that $\Lambda$ is finitely generated as a $K$-module (it is an order) insures that $T(\Lambda) \subset K$. The complementary module, $C = C(\Lambda/K)$, and the Dedekind different, $D = D(\Lambda/K)$, are defined, as usual, as follows:

$$C = \{x \in \Sigma : T(x\Lambda) \subset K\},$$
$$D = \{x \in \Sigma : Cx \subset \Lambda\}.$$

Define the $K$-homomorphism $t: C \to \operatorname{Hom}_K(\Lambda, K)$ by $t(x)(y) = T(xy)$ for all $x \in C$, all $y \in \Lambda$.

PROPOSITION 1. *$t$ is an isomorphism and* $t(D)(1) = T(D(\Lambda/K)) = N(\Lambda/K)$.

OUTLINE OF PROOF. It is shown first that each of the $K$-modules defined behaves nicely under localization at a maximal ideal of $K$. One

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