ON FLAT BUNDLES

BY F. W. KAMBER AND PH. TONDEUR[1]

Communicated by J. Milnor, May 10, 1966

A principal G-bundle ξ on X is *flat* if and only if it is induced from the universal covering bundle of X by a homomorphism $\pi_1 X \to G$ [6, Lemma 1]. First the holonomy map of a principal G-bundle is defined and flat bundles are characterized. Then the reduction problem with respect to a homomorphism $\tau: \Phi \to G$ of a finite abelian group Φ is discussed for $G = O(n)$, $SO(n)$ and $U(n)$.

1. **The holonomy map of a principal bundle.** For a differentiable principal G-bundle ξ on X a connection defines a holonomy map $\Omega X \to G$. The homotopy class of this map is an invariant of ξ, as shown e.g. in [2]. We first give a topological version of this invariant. Let G be a topological group, X a space and ξ a G-bundle with projection $p: T \to X$. EX denotes the space of paths starting from the basepoint of X. Choose a basepoint in T lying in the fiber over the basepoint of X. A section s of the principal EG-bundle $E(p): ET \to EX$ defines a map $h: \Omega X \to G$ as follows. For $\omega \in \Omega X$ there is a unique $h(\omega) \in G$ sending the basepoint of T to the endpoint of $s(\omega)$.

THEOREM 1.1.

(i) *$h: \Omega X \to G$ is an H-map (that is: h carries products into products, up to homotopy).*

(ii) *The equivalence class (under inner automorphisms of G) of the homotopy class of h is an invariant of ξ, called the holonomy map $h(\xi)$ of ξ.*

(iii) *$h(X, G): P(X, G) \to [\Omega X, G]$ defined by $h(X, G)(\xi) = h(\xi)$ is a natural transformation.*

Here $P(X, G)$ denotes the isomorphism classes of numerable G-bundles on X. No distinction is made between a G-bundle and its classifying map $X \to BG$. Then the classification theorem of [3] for numerable bundles over arbitrary spaces can be expressed by $P(X, G) = [X, BG]$.

PROPOSITION 1.2. *For the universal G-bundle η_G the holonomy map $h(\eta_G): \Omega BG \to G$ is a homotopy equivalence.*

2. **Flat bundles.** Let G_d be the underlying discrete group of G and

[1] The first author is a Miller fellow, and the second author was partially supported by National Science Foundation grants GP-1611 and GP-3990.