DISJOINT STEINER SYSTEMS ASSOCIATED WITH THE MATHIEU GROUPS

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A *Steiner system* of type t-d-n is a collection, $\mathfrak{D}$, of subsets of a set S satisfying:

(i) The cardinality of S is n.

(ii) Each subset in $\mathfrak{D}$ has cardinality d.

(iii) Every subset of S of cardinality t is contained in precisely one subset in $\mathfrak{D}$.

Here t, d, and n are positive integers satisfying $t<d<n$.

Two Steiner systems, S, $\mathfrak{D}$ and S', $\mathfrak{D}'$ are called *equivalent* if there is a bijection, $\phi: S\to S'$, such that $\phi(D)\in\mathfrak{D}'$ if and only if $D\in\mathfrak{D}$. If $S=S'$ and $\mathfrak{D}=\mathfrak{D}'$, the set of equivalences forms a group, called the *automorphism group* of the Steiner system S, $\mathfrak{D}$; it is a subgroup of the symmetric group on S. If S, $\mathfrak{D}$ and S, $\mathfrak{D}'$ are Steiner systems of the same type they are said to be *disjoint* if $\mathfrak{D}\cap\mathfrak{D}'$ is empty.

Among the most remarkable Steiner systems are the five associated with the Mathieu groups; i.e., those five Steiner systems whose automorphism groups are the five Mathieu groups. Witt [5] and [6] discussed them in detail, showing that they were unique up to equivalence.

Two of these five systems are of central importance. For if S, $\mathfrak{D}$ is a Steiner system of type t-d-n and X a subset of S of cardinality h with $h<t$, then $S-X$, $\{D\cap(S-X)\mid X\subset D\in\mathfrak{D}\}$ is a Steiner system of type $(t-h)$-$(d-h)$-$(n-h)$. The two central Steiner systems referred to are of types 5-6-12 and 5-8-24. Their automorphism groups are, respectively, the Mathieu groups M_{12} and M_{24}. The other three Steiner systems, of types 4-5-11, 4-7-23, and 3-6-22, are derived from these two by the above method; their automorphism groups are respectively, M_{11}, M_{23}, and M_{22}.

We have found a simple proof of the following

THEOREM. *There exist two disjoint Steiner systems of each of the above five types.*

It suffices to prove this for the two central systems of types 5-6-12 and 5-8-24. The proof, to be published elsewhere, uses a simple relationship between these Steiner systems and certain so-called error-correcting codes. That there are two systems of type 5-6-12 follows