AN ASYMPTOTIC FORMULA FOR THE EIGENVALUES OF THE LAPLACIAN OPERATOR IN AN UNBOUNDED DOMAIN

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F. Rellich [5] and, more generally, A. M. Molcanov [3] have shown that the problem

$$(1)\qquad \begin{aligned} \tfrac{1}{2}\Delta^2 u(x) + \lambda u(x) &= 0, \qquad x \in \Omega \\ u(x) &= 0, \qquad x \in \partial\Omega \end{aligned}$$

has a discrete spectrum (and consequently a complete orthonormal system of eigenfunctions in $\mathcal{L}_2(\Omega)$) provided that Ω is a "quasi-bounded" domain in E_n. A domain Ω is said to be quasi-bounded if it is either bounded or satisfies

$$\lim_{x\to\infty, x\in\Omega} \operatorname{dist}(x, \partial\Omega) = 0.$$

(See [1] for a proof of Molcanov's result, based on a generalization of the Kondrachoff embedding theorem for the Sobolev spaces $H_0^m(\Omega)$.) The problem of determining the asymptotic behavior of the eigenvalues of (1) has remained open (cf. [2, p. 233]).

In the present note we consider the above problem from the point of view of random processes, as described in detail for the case of a bounded domain, as well as for the case of the operator $-\frac{1}{2}\Delta^2 + V(x)$ (with $V(x) \to +\infty$ as $|x| \to \infty$) on an unbounded domain, in the papers of D. Ray [4] and M. Rosenblatt [6]. We will show that if Ω satisfies the following condition

$$(2)\qquad m(\Omega \cap [a < |x| < a+1]) = O(a^{-\beta})$$

for some $\beta > \frac{1}{2}$, then simple modifications of Ray's arguments suffice to prove discreteness of the spectrum, as well as to obtain an asymptotic formula for the eigenvalues.

We take Ray's paper [4] as a starting point. Thus (assuming a cone condition for Ω, as described in Theorem 1 below) we already have a Green's function $K(x, y, t)$ corresponding to the equation

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