NUCLEARITY IN AXIOMATIC POTENTIAL THEORY[1]

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1. **Introduction.** The axiomatic approach to potential theory instituted by Brelot [3] is well known; it abstracts in an elegant manner the properties of harmonic functions which underlie much of classical potential theory and—from a more utilitarian viewpoint—reduces the question of determining whether elliptic equations with certain classes of coefficients have a boundary-value problem theory resembling that of the Laplace equation to a few more tractable questions of an essentially local character. The purpose of this note is to announce that certain properties of the linear spaces of solutions of elliptic equations with highly differentiable coefficients (properties which bear on the construction of kernel functions) are also present in the axiomatic settings, as are certain types of behavior at ideal boundaries. We list only the main results; subsidiary results and proofs will be given elsewhere.

2. **Nuclearity.** Establishment of the nuclearity results does not require the full power of the Brelot axioms. Let W be a locally compact Hausdorff space, and let there be given a set $\mathcal{H}$ of continuous real-valued functions on W satisfying the following (sheaf) axiom:

I. The domains of elements of $\mathcal{H}$ are open subsets of W; each $f\in\mathcal{H}$ is continuous on its domain; for fixed open $\Omega\subseteq W$ the set $\mathcal{H}_\Omega=\{f \mid f\in\mathcal{H}$, Domain $(f)=\Omega\}$ is a real vector space, and a function g with open domain $\Omega\subset W$ belongs to $\mathcal{H}$ iff for each $x\in\Omega$, $\exists h\in\mathcal{H}$ and open ω with $x\in\omega\subseteq\Omega$ such that $g|_\omega=h|_\omega$.

Given two classes $\mathcal{H}$ and $\mathcal{K}$ satisfying I above, we shall say $\mathcal{K}$ is a subclass of $\mathcal{H}$ iff $\mathcal{K}\subseteq\mathcal{H}$; this is equivalent to saying that for any open $\Omega\subseteq W$ the vector space $\mathcal{K}_\Omega$ is a subspace of $\mathcal{H}_\Omega$.

For open $\Omega\subseteq W$ let $\mathcal{H}_\Omega{}^c$ denote the vector subspace of $\mathcal{H}_\Omega$ consisting of those functions which have (necessarily unique) continuous extensions to $\bar{\Omega}$. A relatively compact open subset Ω will be said to be *regular* (with respect to $\mathcal{H}$) if there exists a subset $\delta\Omega\subseteq\bar{\Omega}$ with the property that the restriction mapping $f\to f|_{\delta\Omega}$ of $\mathcal{H}_\Omega^c$ into $\mathcal{C}(\delta\Omega)$ is an order-preserving isomorphism (onto); while the uniqueness of $\delta\Omega$ is not important in what follows, conditions can be given which insure

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