EXTENSIONS OF BRANDT SEMIGROUPS

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The purpose of this note is to announce the determination of all (ideal) extensions of a Brandt semigroup by an arbitrary semigroup with zero and to give two applications of this result. We use the terminology and notation of [1].

THEOREM. *Let* $(V, \circ)$ *be an extension of a Brandt semigroup* S *by an arbitrary semigroup* T *with zero,* $0'$. *Let* S *be given the Rees representation* $S=M^0(G;\ I,\ I;\ \Delta)$. *Then there exists a partial homomorphism* $w: A\rightarrow w_A$ *of* $T^*=(T\backslash 0')$ *into* $\mathcal{J}_I$ *the full symmetric inverse semigroup on* I. *Let* s_A *and* t_A *denote the domain and range of* w_A *respectively. If* $AB=0'$ *(juxtaposition denoting multiplication in* T*) either* $s_A\cap t_B=\square$ *or* $s_A\cap t_B$ *is a single element* $d_{A,B}$. *For each* $A\in T^*$ *there exists a mapping* ψ_A *of* s_A *into the group* G *such that for* $AB=0'$

$$(i\psi_A)(iw_A\psi_B) = i\psi_{AB} \quad \text{for all } i \in s_{AB}.$$

The products in V *are given by*

$$(1)\quad \text{(a) } A\circ B = AB \quad \textit{if } AB = 0' \textit{ in } T,$$

$$\text{(b) } A\circ B = 0 \ (\textit{in } S) \quad \textit{if } AB = 0' \ (\textit{in } T) \textit{ and } t_A\cap s_B = \square,$$

$$\text{(c) } A\circ B = (d_{A,B}\overset{-1}{w_A}\psi_A)(d_{A,B}\psi_B);\ d_{A,B}\overset{-1}{w_A},\ d_{A,B}w_B) \quad \textit{if } AB = 0' \ (\textit{in } T) \textit{ and } t_A\cap s_B = d_{A,B}.$$

$$(2)\quad (a;i,j)\circ A = \begin{cases}(a(j\psi_A);\ i,\ jw_A) & \textit{if } j\in s_A,\\ 0 & \textit{if } j\,\overline{\in}\, s_A,\end{cases}$$

$$0\circ A = 0$$

$$(3)\quad A\circ(a,i,j) = \begin{cases}((i\overset{-1}{w_A}\psi_A)a;\ i\overset{-1}{w_A},\ j) & \textit{if } i\in t_A,\\ 0 & \textit{if } i\,\overline{\in}\, t_A,\end{cases}$$

$$A\circ 0 = 0.$$

Conversely let S *be a Brandt semigroup and* T *be a semigroup with zero such that* $S\cap T=\square$. *If we are given the mappings* w *and* ψ_A *described above and define product* $\circ$ *in the class sum of* S *and* T^* *by* (1)–(3), *then* V *is an extension of* S *by* T.

REMARK. An extension of S by T always exists [1].