ON CERTAIN BISIMPLE INVERSE SEMIGROUPS[1]

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If S is a semigroup, E_S will denote the collection of idempotents of S. A bisimple semigroup S is called I-bisimple if and only if $E_S = \{e_i : i \in I$, the integers$\}$ with $e_i \leqq e_j$ if and only if $i \geqq j$. We announce the determination of the structure of I-bisimple semigroups mod groups and a determination of several of their properties. We also give a certain generalization of the bicyclic semigroup and indicate an application of this result. We use the notation and terminology of [2].

THEOREM 1. *S is an I-bisimple semigroup if and only if $S \cong GXIXI$ under the multiplication*

$$(1) \qquad (g, n, m)(h, p, q) = (g\alpha^{p-r}h\alpha^{m-r}, n + p - r, m + q - r)$$

where $r = \min(m, p)$, α is an endomorphism of G, and α^0 is the identity transformation or equivalently

$$(g, n, m)(h, p, q) = (g\alpha^{s-m-p}h\alpha^{s-q}, n + p, s)$$

where $s = \max(m+p, q)$.

PROOF. [9, Theorem], [1, Main Theorem], [8, Theorem 1.2 and Theorem 2.2] and [5, Theorem 3.3] are important.

REMARK. An I-bisimple semigroup S has no identity and hence its structure may not be obtained by specializing the Clifford structure theorem [1]. S is a union of a chain of bisimple (inverse) semigroups S_i ($i \in I$) with identity such that $E_{S_i} = \{e_i : i \in I^0$, the nonnegative integers$\}$ with $e_i \leqq e_j$ if and only if $i \geqq j$.[2] The structure of these semigroups was given mod groups by Reilly [6] and Warne [11]. Warne obtained the result by specializing the Clifford structure theorem [1]. Incidently, the multiplication is given by (1) with I^0 replaced for I.

If S is an I-bisimple semigroup with structure group G and structure endomorphism α, we will write $S = (G, \alpha)$.

Let N denote the natural numbers.

THEOREM 2. *Let $S = (G, \alpha)$ and $S^* = (G^*, \beta)$. Let $\{f_i : i \in I \backslash N\}$ be a*

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[1] These structure theorems represent a next stage in the development of bisimple semigroups to the Rees Theorem in that the determination is complete (mod groups).

[2] The structure of bisimple (inverse) semigroups such that E_S is linearly ordered has been given mod bisimple inverse semigroups with identity by Warne [9].