RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited. Manuscripts more than eight typewritten double spaced pages long will not be considered as acceptable.

AN INEQUALITY CONCERNING MEASURES

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If μ is a complex measure (countably additive on a σ-field of subsets of some space), it is obvious that there is a measurable set E such that

$$|\mu(E)| \geqq \tfrac{1}{4}\|\mu\|$$

where $\|\mu\|$ denotes the total variation of μ. In fact a set E can be found for which

$$|\mu(E)| \geqq \frac{1}{\pi}\|\mu\|.$$

We shall give a simple proof of this. If μ is a vector valued measure with values in R^n (with the usual Euclidean norm) we shall show by a suitable modification of our argument that there is a set E with

$$\|\mu(E)\| \geqq \frac{1}{2\pi^{1/2}} \frac{\Gamma(n/2)}{\Gamma((n+1)/2)}\|\mu\|.$$

Asymptotically this is $\|\mu\|/(2\pi n)^{1/2}$, which is much better than the obvious $\|\mu\|/2n$.

THEOREM 1. *Let μ be a complex valued measure of total variation* 1. *Then there is a measurable set E such that* $|\mu(E)| \geqq 1/\pi$.

PROOF. Consider first the special case where μ is a Borel measure on the unit circle of the complex plane (which we identify with the real line (mod 2π)), and is such that for every measurable set E,

$$\mu(E) = \int_E e^{i\theta}\,|\mu|\,(d\theta)$$

where $|\mu|(E)$ denotes the total variation of μ on the set E. Then

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