SELF-EQUIVALENCES OF $(n-1)$-CONNECTED $2n$-MANIFOLDS[1]

BY P. J. KAHN

Communicated by J. Milnor, Janauary 31, 1966

1. Introduction and statement of main results. All spaces have basepoints, and all maps of spaces are basepoint-preserving. A self-equivalence of a space X is a homotopy class of homotopy equivalences $X \to X$. Map-composition induces an operation on the set of self-equivalences of X, making it into a group, $\mathcal{E}(X)$.

Arkowitz and Curjel [1] and Weishu Shih [7] have obtained certain general results about $\mathcal{E}(X)$ by studying the Postnikov decomposition of X. More recently P. Olum [5] presented an explicit computation of $\mathcal{E}(X)$ in the case that X is a pseudo-projective plane.

Our results concern the structure of $\mathcal{E}(X)$ in the case that X is a closed, compact, oriented, C^∞, $(n-1)$-connected $2n$-manifold, $n \geqq 2$. We place these restrictions on X throughout the rest of this paper. Our methods are dual to those of [1] and [7] in the sense that we proceed by examining a cell-decomposition of X.

A word about notation: X_n is the n-skeleton of X in some fixed, minimal CW-decomposition of X, SX_n is its suspension, and $\pi(SX_n, X)$ is the group of homotopy classes of maps $SX_n \to X$.

THEOREM 1. *There is an exact sequence,*

$$\pi(SX_n, X) \xrightarrow{(Sb)^* + \bar{\psi}} \pi_{2n}(X) \xrightarrow{\rho} \mathcal{E}(X) \xrightarrow{R} \mathcal{E}(X_n),$$

the homomorphisms of which will be described in §2.

It is easy to show that $\pi_{2n}(X)$ is finite.

COROLLARY TO THEOREM 1. *Kernel R is finite.*

X_n is a one-point union of (at least two) n-spheres, so that $H_n(X_n) = H_n(X)$ is finitely generated free abelian. Moreover, it is easy to show that the homology functor H_n takes $\mathcal{E}(X_n)$ isomorphically onto the group of automorphisms of $H_n(X)$. We call this automorphism group $\mathrm{Aut}(H_n(X))$.

Let $\mu: H_n(X) \otimes H_n(X) \to Z$ be the integral bilinear form determined by the intersection pairing on $H_n(X)$. Wall [8] shows that μ, together with a certain function $H_n(X) \to \pi_{2n-1}(S^n)$, completely deter-

[1] This work was partly supported by NSF grant GP 3685.