MICROBUNDLES AND BUNDLES

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The following concerns a generalization of the Kister-Mazur representation theorem for microbundles, which says that any microbundle over a locally finite, finite dimensional simplicial complex contains a bundle, unique up to bundle isomorphism. More precisely, the purpose of this note is to prove the following:

MICROBUNDLE REPRESENTATION THEOREM. (a) *Let* $\mu: X \xrightarrow{s} E \xrightarrow{p} X$ *be an* R^q*-microbundle over a paracompact base space, and let* $U \subset X$ *be a neighbourhood of a closed set* $A \subset X$. *Suppose* $\mu | U$ *is actually an* R^q*-bundle. Then there is a neighbourhood* E' *of* sX *in* E *and a neighbourhood* U' *of* A *in* X *such that* $X \xrightarrow{s'} E' \xrightarrow{p'} X$ *is an* R^q*-bundle* ξ (*where* $i \circ s' = s$, $p' \circ i = p$, $i: E' \subset E$) *and* $\xi | U' = \mu | U'$.

(b) *Suppose* ξ_1, ξ_2 *are* R^q*-bundles contained in* μ *and that* $\xi_1 | U' = \xi_2 | U'$ *for some neighbourhood* U' *of* A *in* X. *Then there is a bundle isomorphism* $\xi_1 \approx \xi_2$ *which is the identity over* A.

A proof of the representation theorem will be outlined after some preparatory work. It depends strongly on the germ extension theorem for trivial bundles (Theorem 1 below). This result is stated in Mazur [3] but seems false unless some restrictions are placed on the base space (or the germ). In the case where X is paracompact it seems to follow from the general theory of dilation neighbourhoods as developed in [3]. In any case a direct proof is indicated below. It uses methods of Kister and Mazur generalized from the case where X is a simplex to the case where X is any topological space. Reportedly Mazur has used his theory of dilation neighbourhoods to establish the representation theorem in the case where X is locally compact, normal and Lindelöf. Since any such space is of course paracompact his result is contained in ours.

The main results of this paper can be generalized to the case of numerable microbundles; cf. [1]. A more general and detailed version will appear elsewhere.

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1. In the sequel we use the concepts and notations of Milnor [4] except for the following modifications. Instead of an *isomorphism-*

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