INTEGRAL REPRESENTATION OF MULTILINEAR CONTINUOUS OPERATORS FROM THE SPACE OF LEBESGUE-BOCHNER SUMMABLE FUNCTIONS INTO ANY BANACH SPACE

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In [3] was found a representation of linear continuous operators h from the space $L(v, Y)$ of Lebesgue-Bochner summable functions into any Banach space W. The above result was generalized to the case of the space $L_p(v, Y)$ in [4].

In this paper it will be shown that by means of the integral $\int u(f_1, \cdots, f_k, d\mu)$ one can represent multilinear continuous operators from the space

$$L(v_1, Y_1) \times \cdots \times L(v_k, Y_k)$$

into any Banach space. We use the notation of [2] and [8].

Some representations of linear continuous functionals and operators on the space $L_p(v, Y)$ were obtained by Dieudonné [11] and Dinculeanu [12].

The problem of representation of linear continuous operators from the space $C(X, Y)$ of all continuous functions on a compact space X into a Banach space Y, or more generally into a locally convex space, is closely related to the above problem. The classical result of Riesz [24] was generalized by Dunford and Schwartz [15], Foias and Singer [17], Dinculeanu [13], and Swong [28]. The most general results in this field belong to Dinculeanu [13] and Swong [28]. Some new proofs of the results of Dinculeanu [13] one can find in Bogdanowicz [7].

Let (X, V, v) be the product of volume spaces (X_j, V_j, v_j) $(j=1, \cdots, k)$. Denote by H the space of all multilinear continuous operators from the product of the spaces $L(v_j, Y_j)$ $(j=1, \cdots, k)$ into the Banach space W.

THEOREM 1. *To every k-linear continuous operator h from the product of the spaces $L(v_j, Y_j)$ $(j=1, \cdots, k)$ into the Banach space W corresponds a unique vector volume $\mu \in M(v, W_0)$ where $W_0 = L(Y_1, \cdots, Y_k; W)$ such that*

$$h(f_1, \cdots, f_k) = \int u(f_1, \cdots, f_k, d\mu)$$

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