ON THE SPECTRUM OF GENERAL SECOND ORDER OPERATORS[1]

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Let λ_1 be the lowest eigenvalue of the membrane problem

$$\Delta u + \lambda u = 0 \quad \text{in } D,$$
$$u = 0 \quad \text{on } \partial D.$$

It was shown by Barta [1] that if $w > 0$ in D, then

$$\lambda_1 \geqq \inf\left[-\frac{\Delta w}{w}\right].$$

This result has been extended to other selfadjoint problems for second order operators. See [2], [3], and [6].

The purpose of this note is to show that the same technique locates the spectrum of a nonselfadjoint problem in a half-plane. Such a result is of interest in investigating stability, where one needs to know whether there is any spectrum in the half-plane $\operatorname{Re}\lambda \leqq 0$.

In a bounded domain D we consider the differential equation

$$\begin{aligned} L[u] + \lambda k u \equiv \sum_{i,j=1}^{n} a^{ij}(x)\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{1}^{n} b^i(x)\frac{\partial u}{\partial x_i} + c(x)u + \lambda k(x)u \\ = -k(x)f(x) \end{aligned} \tag{1}$$

where $x \sim (x_1, \cdots, x_n)$. The matrix $a^{ij}(x)$ is symmetric and positive definite, $k(x)$ is positive, and all the coefficients are real and bounded in D. However, they need not be continuous.

The boundary ∂D is divided into two disjoint parts Σ_1 and Σ_2, and the boundary conditions are

$$\begin{aligned} u &= 0 \quad \text{on } \Sigma_1, \\ M[u] &\equiv \sum_{1}^{n} e^i(x)\frac{\partial u}{\partial x_i} + g(x)u = 0 \quad \text{on } \Sigma_2. \end{aligned} \tag{2}$$

The vector field $\boldsymbol{e}$ points outward from D.

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