RESEARCH PROBLEMS

3. Richard Bellman: *Singular differential and the Lagrange expansion.*

Consider the differential equation $u=g(t)+\epsilon h(u, du/dt)$, where h is analytic in its arguments, and suppose that we iterate, $u=g+\epsilon h(g, g')+\epsilon^2 h_1+\cdots$, a formal power series in ϵ. Is there an extension of the Lagrange expansion theorem which permits us to obtain the coefficients in a reasonably simple fashion? (Received October 4, 1965.)

4. Richard Bellman: *Lagrange expansion for functionals.*

The partial differential equation $u_t=uu_x$, $u(x, 0)=g(x)$, which has the solution $u=g(x+tu)$ can be used to derive the Lagrange expansion of the solution of $v=c+\epsilon h(v)$ as a power series in ϵ. Can one find an analogous solution of the functional partial differential equation $u_t=u(\delta u/\delta f)$, $u(f, 0)=g(f)$, where $g(f)$ is a given functional of f, and $\delta u/\delta f$ denotes the Gateaux derivative of the functional $u(f, t)$, which can be used to find a Lagrange expansion of the solution of $v=c(x)+\epsilon h(v)$, where h is a functional of v? (Received October 4, 1965.)

5. J. M. Gandhi: *The number of representations of a number as a sum of ten squares.*

Let $\gamma_{2s}(n)$ denote the number of representations of a number n as a sum of $2s$ squares. We shall not discuss representations of n as a sum of an odd number of squares. As usual we call a function $f(n)$ multiplicative if it satisfies the condition

$$f(mn) = f(m)f(n), \qquad (m, n) = 1.$$

It is observed that $\gamma_{2s}(n)$ is not multiplicative for all values of $2s$. For example $\gamma_6(n)$, $\gamma_{24}(n)$, etc. are not multiplicative. But $\gamma_{2s}(n)$ is multiplicative for some values of s. For example it was shown by Gupta and Vaidhya [2] that $\gamma_2(n)/4$ is multiplicative. Since $\sigma(n)$, the sum of the divisors of n and $\sigma_0(n)$, sum of the odd divisors of n are multiplicative and

$$\gamma_4(2n+1) = 8\sigma(2n+1),$$
$$\gamma_4(2n) = 24\sigma_0(n),$$

[**3**, p. 132]

hence $\gamma_4(2n+1)/8$ and $\gamma_4(2n)/24$ are multiplicative. Also

$$\gamma_8(n) = (-1)^{n-1}16\Sigma_d(-1)^{d-1}d^3$$

[**1**, p. 315].