SOME RESULTS ON DIFFERENTIABLE ACTIONS

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In this note, we shall announce some results on differentiable actions of SO(n), SU(n) and Sp(n) on manifolds. Since the detailed proofs are too long to be included here, we shall publish them elsewhere.

THEOREM 1. *Let* ϕ *be a differentiable action of* SO(n), (SU(n), Sp(n)) *on an* m-dim *manifold* M^m *where* $n \geqq 11$ *and* $m \leqq (n-1)^2/4$ ($n \geqq 8$ *and* $m \leqq (n-1)^2/2$, $n \geqq 8$ *and* $m \leqq (n-1)^2$). *If the first rational Pontrjagin class of* M^m, $P_1(M^m)$, *vanishes, then the identity component of any isotropy subgroup,* $(G_x)_0$ *for* $x \in M^m$ *is always conjugate to* SO(k), (SU(k), Sp(k)) *under the standard inclusion for some* $k \geqq \frac{2}{3}n$.

THEOREM 2. *For a given differentiable action* ϕ *of* SO(n), (SU(n), Sp(n)) *on a homotopy sphere* Σ^m (*respectively Euclidean space* R^m, *respectively disc* D^m) *where* $n \geqq 11$ *and* $m \leqq (n-1)^2/4$ ($n \geqq 8$ *and* $m \leqq (n-1)^2/2$, $n \geqq 8$ *and* $m \leqq (n-1)^2$), *we have that*

(i) *all orbits are real* (*complex, quaternionic*) *Stiefel manifolds,*

(ii) *if* SO(n)/SO(k), (SU(n)/SU(k), Sp(n)/Sp(k)) *is the principal orbit and* F *is the fixed point set, then*

$$H^*(F; A) \simeq H^*(S^\gamma; A)$$
$$(\textit{respectively } H^*(F; A) \simeq H^*(R^\gamma; A)$$
$$\textit{respectively } H^*(F; A) \simeq H^*(D^\gamma; A))$$

where

$$\begin{aligned} \gamma = \dim F &= m - n(n-k) \quad \textit{for the } \mathrm{SO}(n) \textit{ case} \\ &= m - 2n(n-k) \quad \textit{for the } \mathrm{SU}(n) \textit{ case} \\ &= m - 4n(n-k) \quad \textit{for the } \mathrm{Sp}(n) \textit{ case} \end{aligned}$$

and

$$\begin{aligned} A &= Z_2 \quad \textit{for the } \mathrm{SO}(n) \textit{ case } (n \textit{ odd}) \\ &= Z \quad \textit{for the other cases.} \end{aligned}$$

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