A CHARACTERIZATION OF THE EUCLIDEAN SPHERE

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1. **Introduction.** Let M be a connected Riemannian manifold of dimension n, $C_0(M)$ its largest connected group of conformal transformations and $I_0(M)$ its largest connected group of isometries. In an earlier paper [2], one of the authors and S. Kobayashi established the following result:

THEOREM 1. *A compact homogeneous Riemannian manifold for which* $C_0(M) \neq I_0(M)$ *and* $n > 3$ *is globally isometric with a sphere.*[2]

In the final step of the proof of this theorem the following statement, which is by no means easy to establish, was utilized:

PROPOSITION 1 (YANO-NAGANO [6]). *A complete Einstein space for which* $C_0(M) \neq I_0(M)$ *and* $n > 2$ *is globally isometric with a sphere.*

Without this fact it was shown that *the simply connected Riemannian covering of M is globally isometric with a sphere.* Using this statement, an elementary proof of Theorem 1, i.e. a proof which does not use Proposition 1, is given (see Proposition 4).

All other results in this direction employ Proposition 1 in the final analysis. We list several of these:

PROPOSITION 2 (NAGANO [4]). *A complete Riemannian manifold with parallel Ricci tensor for which* $C_0(M) \neq I_0(M)$ *and* $n > 2$ *is globally isometric with a sphere.*

This generalizes Proposition 1.

PROPOSITION 3 (LICHNEROWICZ [3]). *Let M be a compact Riemannian manifold of dimension* $n > 2$ *whose scalar curvature R is a positive constant and for which* trace $Q^2 = \text{const.}$ *where Q is the Ricci operator (see* [1, *p.* 87]). *Then, if* $C_0(M) \neq I_0(M)$, *M is globally isometric with a sphere.*

This generalizes Theorem 1 and Proposition 2.

In §4, Proposition 1 will be generalized. Denote the Lie algebra of

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[2] The first part of the proof of Theorem 1 appears in a previous paper published in the Amer. J. Math 84 (1962), 170–174 by S. I. Goldberg and S. Kobayashi entitled *The conformal transformation group of a compact Riemannian manifold.*