

INTEGRAL REPRESENTATIONS OF HERMITIAN FORMS OVER LOCAL FIELDS

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The purpose of this note is to state the solution to the problem of local integral representation of hermitian forms when the characteristic is not 2. Hermitian and quadratic forms are two of the three main types of reflexive forms (cf. [1]). Hermitian theory of forms often follows quickly once the corresponding quadratic theory is known. However, the local integral representation theory for quadratic forms is still incomplete and finding a general solution there appears to be very difficult.

Several results concerning these questions have been obtained over rings and fields of arithmetic type. Questions of fractional equivalence and representation as well as local integral equivalence have been solved for both quadratic and hermitian forms (cf. [2], [3], [4], [7]). Partial solutions to the problem of local integral representation of quadratic forms have been obtained by O'Meara [6] and Riehm [8]. Global results on the equivalence of hermitian forms have been obtained by Shimura [9].

The solution to the question of local integral representation of hermitian forms is obtained through three cases. We are given a local field E (of characteristic $\neq 2$) with a nontrivial involution $*$. Let F be the fixed field of $*$. Then the three cases considered are those where the field extension E/F is (1) unramified, (2) ramified with E nondyadic and (3) ramified with E dyadic. The solutions in the first two cases are identical and can be stated quite easily. The third case is more complicated and some added notation will be necessary. Detailed proofs of these results will appear elsewhere.

1. Notation. Let $| \cdot |$ denote the valuation on E . Let \mathfrak{O} be the ring of integers in E , \mathfrak{U} the group of units and p a prime element of E . The norm and trace of an element of E are defined as usual; set $\pi = N(p)$. It is possible to write $E = F(\sqrt{\theta})$ where θ has one of the following three forms:

$$(1) \quad \theta = \begin{cases} 1 + 4\delta, \\ 1 + \pi^{2k+1}\delta, \\ \pi\delta, \end{cases}$$

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