A NOTE ON LINK GROUPS

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1. **Introduction.** The purpose of this paper is to provide a sufficient geometric condition in order for a link group to be mapped homomorphically onto a free group of rank $r \leqq \mu$, where μ is the number of components of the link. Applying this together with a construction of L. Neuwirth [**3**], we obtain an extension of two results in [**3**] concerning free subgroups of link groups and solvability of link groups.

2. **Group presentations.** By a surface S of type (p, μ, r) we shall mean the disjoint union in S^3 of r tame, orientable, compact, connected 2-manifolds, each with nonempty boundary, where μ is the number of boundary components, and p is the sum of the genera of the components of S. R. H. Fox describes in [**1**] a method of obtaining for any surface of type $(p, 1, 1)$ a model S of the same embedding type which consists of a 2-cell with a number of bands attached which may be made to "lie flat" so that only one side of S is visible. This method may also be applied to a surface of type $(p, \mu, 1)$ and hence to a surface of type (p, μ, r), progressing componentwise.

If $L \subset S^3$ is a link with μ components and S is a surface of type (p, μ, r) with boundary L, then a flat model of S determines a regular projection of L from which we obtain an over presentation [**2**]

$$P_L = (x_1, \cdots, x_m : r_1, \cdots, r_m)$$

of $\pi_1(S^3 - L)$. Also, we can get a presentation

$$P_S = (z_1, \cdots, z_q : s_1, \cdots, s_v)$$

of $\pi_1(S^3 - S)$, where each z_i circles a band just once, and the relators occur where the bands cross each other. Each z_i has the form $x_{i_1} x_{i_2}^{-1}$.

3. **Main results.** Let $L \subset S^3$ be a link with μ components and genus p_0, $G = \pi_1(S^3 - L)$, F_r = free group of rank r.

THEOREM 1. *If L bounds a surface S of type (p, μ, r), then there is an epimorphism $\phi : G \to F_r$.*

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