INTERSECTIONS OF COMBINATORIAL BALLS AND OF EUCLIDEAN SPACES

BY L. C. GLASER

Communicated by O. G. Harrold, August 9, 1965

1. **Introduction.** Poenaru [4] and Mazur [3] gave the first examples of pseudo 4-cells whose products with the unit interval were combinatorial 5-cells. Curtis [1] and Glaser [2] gave similar examples for $n \geqq 5$. In addition, it was shown in [2], that, for $n \geqq 5$, the pseudo n-cell M^n could be expressed as the union of two combinatorial n-cells whose intersection is also a combinatorial n-cell. Unfortunately, the techniques used in [2] gave no hope of lowering the result to $n=4$.

The purpose of this announcement is to give another example of a pseudo 4-cell W with the property that $W \times I \approx I^5$, but in addition W can be expressed as the union of two combinatorial 4-cells whose intersection is also a combinatorial 4-cell. This also gives an example of two Euclidean 4-spaces intersecting in an Euclidean 4-space so that the union is not topologically E^4.

Our techniques and terminology basically follow that found in [6], [7].

2. **Construction.** Let us consider a figure eight expressed as the union of four line segments α, β, γ, and δ and three vertices a, b, and c as indicated in Figure 1. Let K be the contractible noncollapsible 2-complex formed by attaching two disks to the figure eight by the formula $\beta\gamma\gamma^{-1}\delta^{-1}\delta\alpha$ and $\delta\alpha\alpha^{-1}\beta^{-1}\beta\gamma$.

Let T be a solid two-holed 3-dimensional torus in E^3. The pseudo 4-cell W will be formed by attaching two 2-handles to the boundary of $T \times [0, 1]$ along the curves Γ_1 and Γ_2 embedded in $\operatorname{int}(T \times \{1\}) \subset T \times [0, 1]$ as indicated in Figure 1.

LEMMA 1. *W can be considered as a regular neighborhood of a combinatorial embedding of K in W.*

PROOF. We can obtain a copy of K by "pushing" $\Gamma_1 \cup \Gamma_2$ down to a core of $T \times [0, 1]$ in an appropriate manner and then considering the point set consisting of the trace of this "pushing" plus the two disks gotten from adding the two 2-handles. The remainder of the proof consists of showing how we can triangulate W so that K is a subcomplex and $W \searrow K$. The techniques used here are similar to those of [7].