RINGS OF MEROMORPHIC FUNCTIONS

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1. Introduction. This paper concerns itself with certain rings of meromorphic functions on noncompact Riemann surfaces. Let Ω denote a noncompact Riemann surface. We denote by A the collection of all mappings of Ω into the complex plane C which are analytic on Ω. Also, we denote by M the collection of all mappings of Ω into the Riemann sphere Σ which are meromorphic on Ω. As is well known, A is an integral domain under the operations of pointwise addition and multiplication, and M is the field of quotients of A. The rings considered here are those subrings of M which contain the ring A. Such subrings will be referred to as *A-rings of M*. In particular, A is itself an A-ring of M, as is the field M.

The ring A has been extensively investigated in recent years, and a considerable amount of information concerning the ideal theory of this ring has been obtained. The main result here is the theorem of Helmer [3], which asserts that every finitely generated ideal of A is actually a principal ideal of A. This theorem is the basis for most of the known results on the ideal theory of A, as is evident from the papers of Henriksen [4], [5], Kakutani [7], and Banaschewski [2].

We announce some results pertaining to the A-rings of M, the principal one of which is a characterization of these rings (Theorem 3). Thanks to this characterization, a number of theorems concerning the ideal theory of A extend to any A-ring of M, as, for example, the theorem of Helmer. Inasmuch as A is itself an A-ring, our results may be considered as generalizations of the corresponding results for A.

The methods involved in the proofs of these results involve a study and exploitation of the valuation theory of M, which was previously considered by Alling [1]. In particular, we make considerable use of the valuation rings of M which are also A-rings of M. These rings are readily identified by means of Helmer's theorem, and they may be employed to prove many of the known results on the ideal theory of A. Moreover, the arguments involved in these proofs frequently apply to any A-ring of M. It is also possible to classify certain A-rings by these methods, and we are able, for example, to determine the noetherian A-rings of M.

Finally, we consider the extent to which a Riemann surface is determined by its A-rings. More exactly, we can show that if two A-